Experiment 1

AIM: Study of Lissajous Figures.

Apparatus Required:

- **Lissajous Pattern Trainer**
- Patch Cords.
- Power Cable.

Theory:

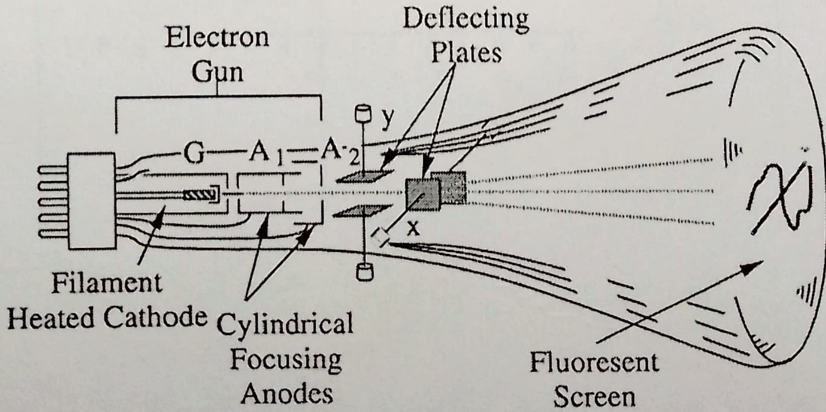
Lissajous figure, also called BOWDITCH CURVE, pattern produced by the intersection of two sinusoidal curves the axes of which are at right angles to each other. First studied by the American mathematician Nathaniel Bowditch in 1815, the curves were investigated independently by the French mathematician Jules-Antoine Lissajous in 1857–58. Lissajous used a narrow stream of sand pouring from the base of a compound pendulum to produce the curves.

If the frequency and phase angle of the two curves are identical, the resultant is a straight line lying at 45° (and 225°) to the coordinate axes. If one of the curves is 180° out of phase with respect to the other, another straight line is produced lying 90° away from the line produced where the curves are in phase (*i.e.*, at 135° and 315°).

Otherwise, with identical amplitude and frequency but a varying phase relation, ellipses are formed with varying angular positions, except that a phase difference of 90° (or 270°) produces a circle around the origin. If the curves are out of phase and differing in frequency, intricate meshing figures are formed.

Of particular value in electronics, the curves can be made to appear on an oscilloscope, the shape of the curve serving to identify the characteristics of an unknown electric signal. For this purpose, one of the two curves is a signal of known characteristics. In general, the curves can be used to analyze the properties of any pair of simple harmonic motions that are at right angles to each other.

Cathode Ray Oscilloscope (CRO) is very important electronic device. CRO is very useful to analyze the voltage wave form of different signals. The main part of CRO is CRT (**Cathode Ray Tube**). A simple CRT is shown in figure below-



When both pairs of the deflection plates (horizontal deflection plates and vertical deflection plates) of CRO (**Cathode Ray Oscilloscope**) are connected to two sinusoidal voltages, the patterns appear at CRO screen are called the **Lissajous pattern**.

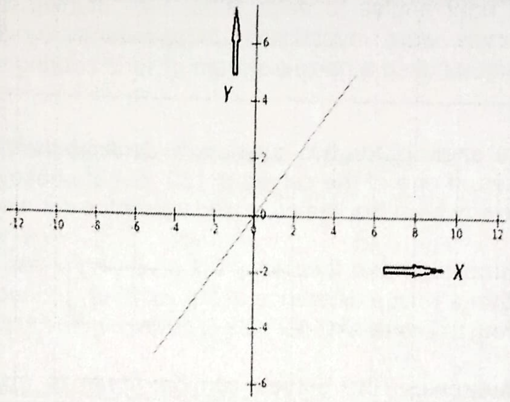
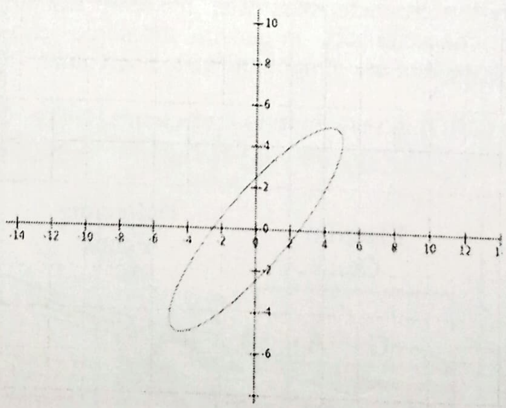
Shape of these **Lissajous pattern** changes with changes of phase difference between signal and ration of frequencies applied to the deflection plates (traces) of **CRO**. Which makes these **Lissajous patterns** very useful to analysis the signals applied to deflection plated of CRO. These lissajous patterns have two Applications to analysis the signals. To calculate the phase difference between two sinusoidal signals having same frequency. To determine the ratio frequencies of sinusoidal signals applied to the vertical and horizontal deflecting plates.

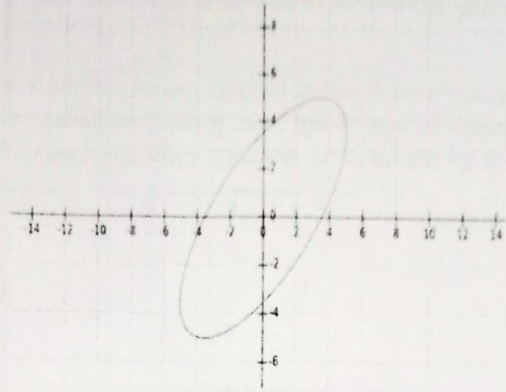
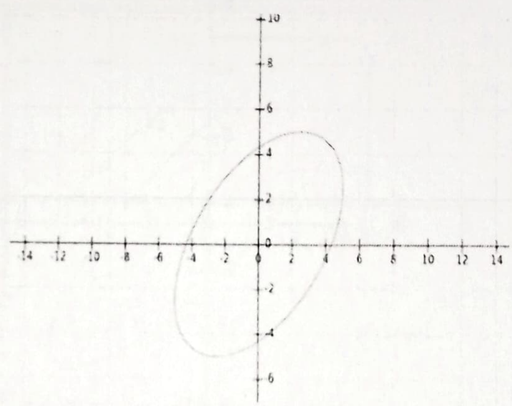
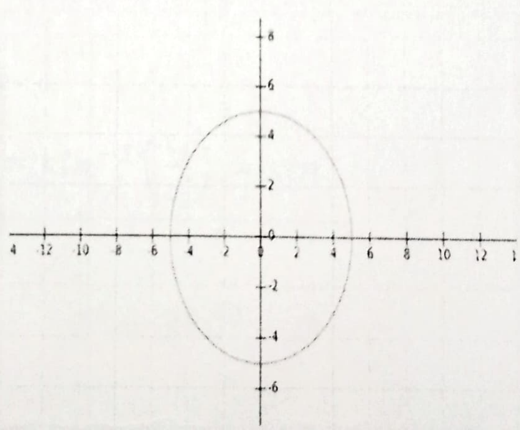
Calculation of the phase difference between two Sinusoidal Signals having same frequency

When two sinusoidal signals of same frequency and magnitude are applied two both pairs of deflecting plates of **CRO**, the Lissajous pattern changes with change of phase difference between signals applied to the CRO.

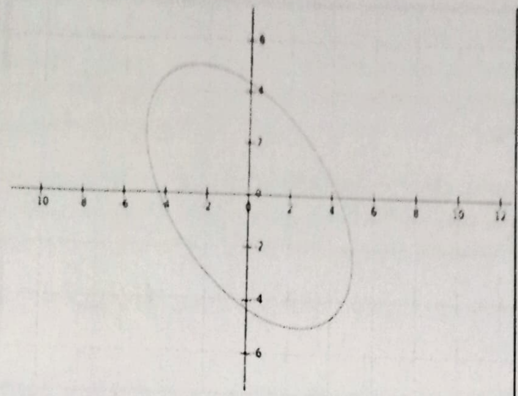
For different value of phase differences, the shape of Lissajous patterns is shown in figure below,

Table 1.1

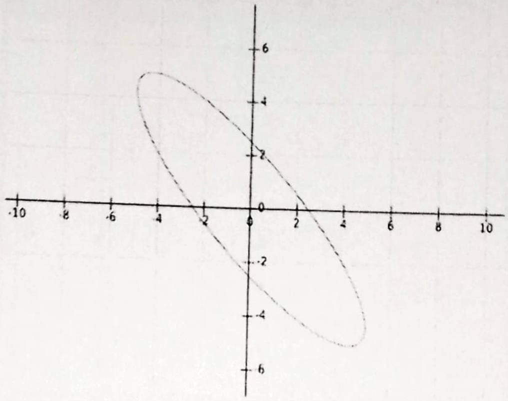
S. No.	Phase angle difference 'ø'	Lissajous Pattern appeared at CRO Screen
1	0° & 360°	
2	30° or 330°	

3	45° or 315°	
4	60° or 300°	
5	90° or 270°	

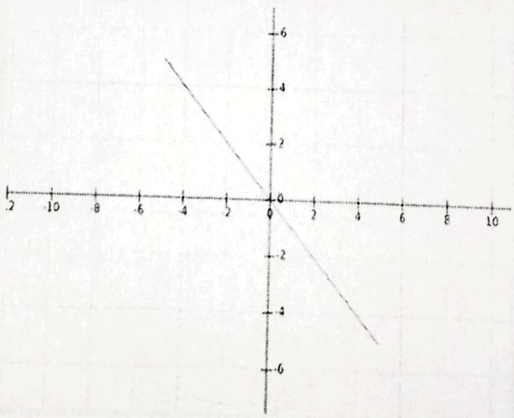
6 120° or 240°



7 150° or 210°

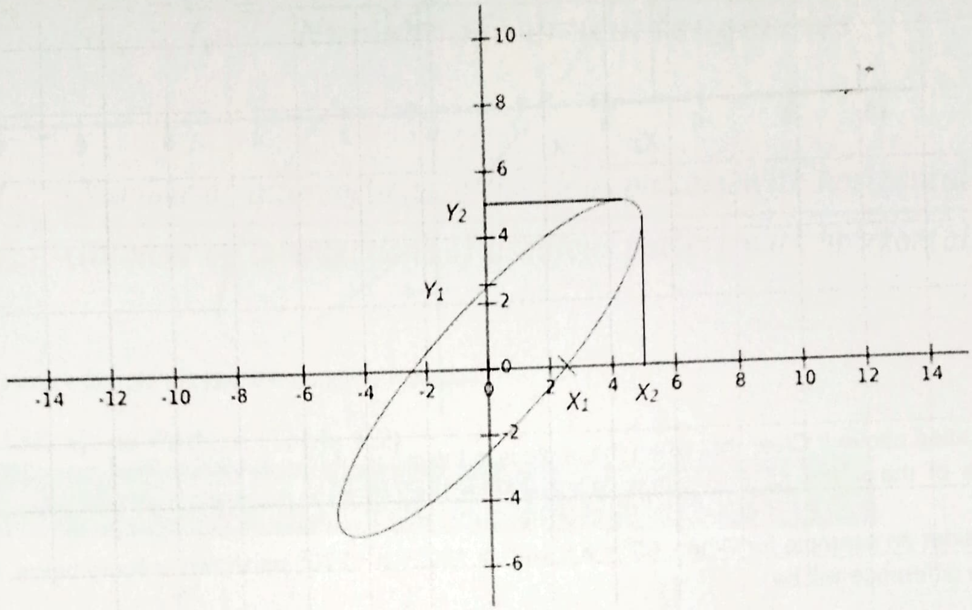


8 180°



There are two cases to determine the phase difference ø between two signals applied to the horizontal & vertical plates,

Case - I: When, 0 < ø < 90° or 270° < ø < 360° : - As we studied above it clear that when the angle is in the range of 0 < ø < 90° or 270° < ø < 360°, the Lissajous pattern is of the shape of Ellipse having major axis passing through origin from first quadrant to third quadrant: Let's consider an example for 0 < ø < 90° or 270° < ø < 360°, as shown in figure below



In this condition the phase difference will be,

$$\emptyset = sin^{-1}\left(\frac{x_1}{x_2}\right) = sin^{-1}\left(\frac{y_1}{y_2}\right)$$

Another possibility of phase difference,

$$\emptyset' = 360^0 - \emptyset$$

From Above given Lissajous pattern

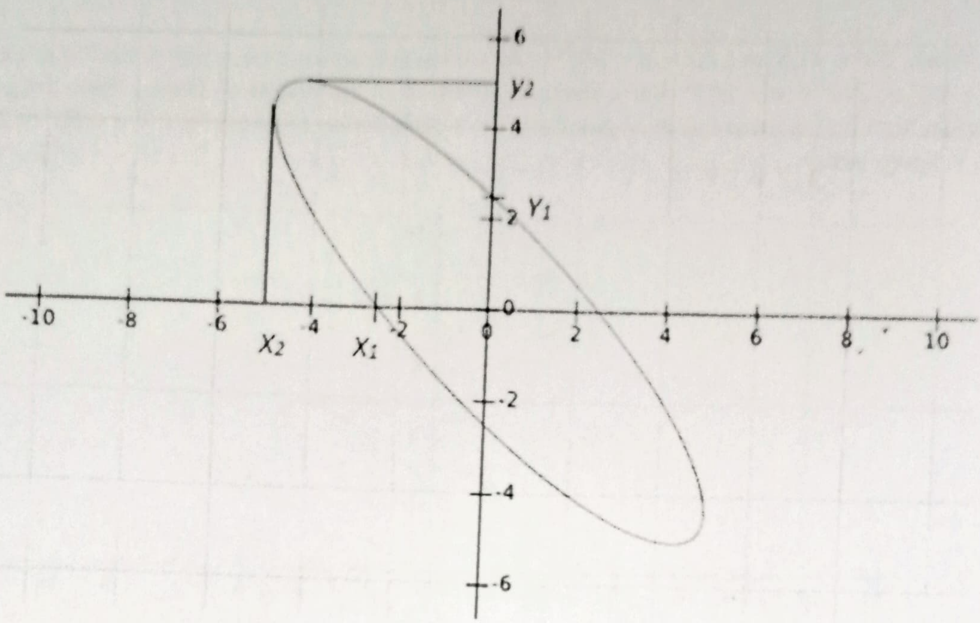
$$x_1 = 2.25 \ \& \ x_2 = 4.5$$

$$\text{Hence, } \emptyset = sin^{-1}\left(\frac{x_1}{x_2}\right) = sin^{-1}\left(\frac{2.25}{4.5}\right) = 30^0$$

Another Possibility of Phase Difference.

$$\emptyset' = 360^0 - \emptyset = 360^0 - 30^0 = 330^0$$

Case - II: When 90° < ø < 180° or 180° < ø < 270°



As we studied above it Clear that when the angle is in the range of 0° < ø < 90° or 270° < ø < 360°, the Lissajous Pattern is of the shape of Ellipse having major axis passing through origin from second quadrant to fourth quadrant:

Let's consider an example for When, 90° < ø < 180° or 180° < ø < 270°, as shown in figure below, In this condition the phase difference will be,

$$\emptyset = 180^0 - sin^{-1}\left(\frac{x_1}{x_2}\right) = 180^0 - sin^{-1}\left(\frac{y_1}{y_2}\right).$$

Another possibility of phase difference,

$$\emptyset' = 360^0 - \emptyset$$

From Above given Lissajous pattern

$$x_1 = 2.25 \;\&\; x_2 = 4.5$$

$$\text{Hence, } \emptyset = 180^0 - sin^{-1}\left(\frac{x_1}{x_2}\right) = 180^0 - sin^{-1}\left(\frac{2.25}{4.5}\right) = 180^0 - 30^0 = 150^0$$

Another Possibility of Phase Difference,

$$\emptyset' = 360^0 - \emptyset = 360^0 - 150^0 = 210^0$$

To determine the ratio of frequencies of signal applied to the vertical and horizontal deflecting plates: To determine the ratio of frequencies of signal by using the Lissajous pattern, simply draw arbitrary horizontal and vertical line on lissajous pattern intersecting the Lissajous pattern. Now count the number of horizontal and vertical tangencies by Lissajous pattern with these horizontal and vertical line. Then the ratio of frequencies of signals applied to deflection plates,

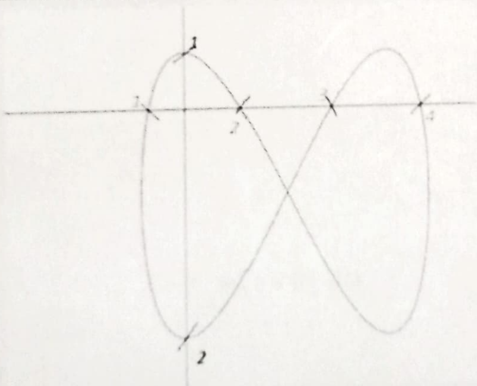
$$\frac{\omega_y}{\omega_x} = \frac{f_y}{f_x} = \frac{Number\ of\ horizontal\ tangencies}{Number\ of\ vertical\ tangencies}$$

Or

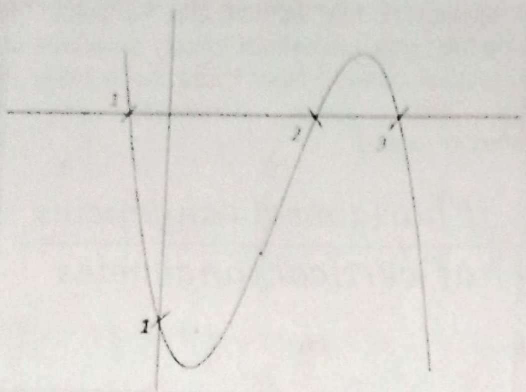
$$\frac{\omega_y}{\omega_x} = \frac{f_y}{f_x} = \frac{(Number\ of\ intersections\ of\ lissajous\ pattern\ with\ horizontal\ line)}{(Number\ of\ intersections\ of\ lissajous\ pattern\ with\ horizontal\ line)}$$

Let consider some example to clear the concept in details:

Examples Lissajous Pattern (Table 1.2)

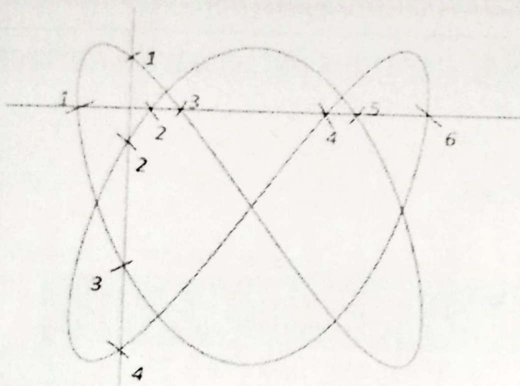
S. No.	Lissajous Pattern	Ratio of Frequencies f_y/f_x
1		$f_y/f_x = 4/2 = 2$

2



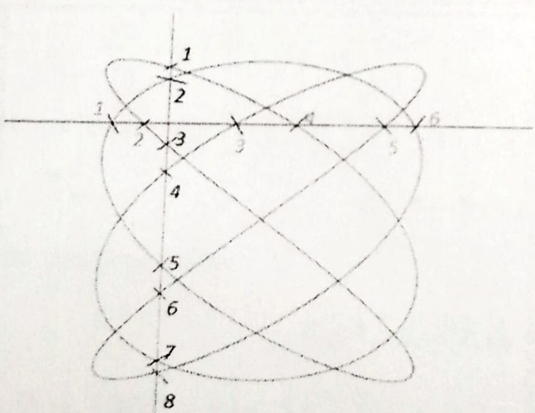
$f_y/f_x = 3/1 = 3$

3



$f_y/f_x = 6/4 = 3/2$

4



$f_y/f_x = 6/8 = 3/4$