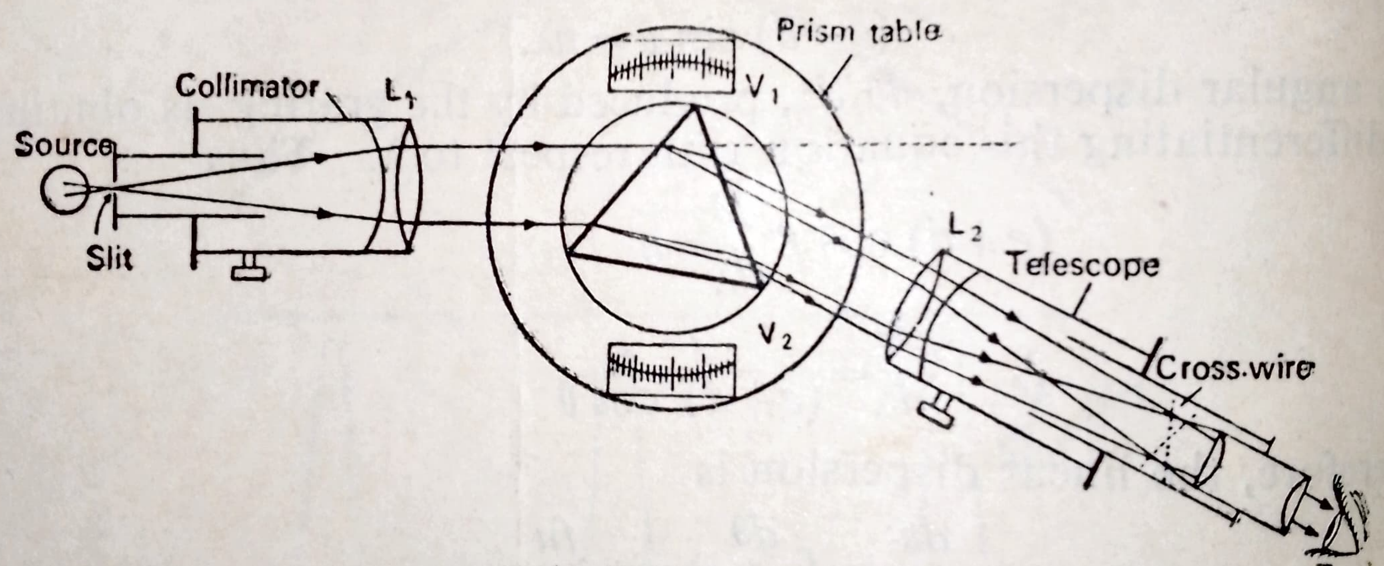
Object. To draw normal dispersion curve by using a spectrometer and determine the Cauchy's constants.

Apparatus : Spectrometer, mercury lamp, prism, diffraction grating, spirit-level, reading lens, reading lamp, etc.

Spectrometer : It is an important laboratory instrument used for the study of spectra produced by prisms and gratings, for the measurement of dispersive powers, wavelengths of spectral lines and the refractive indices of solids and liquids

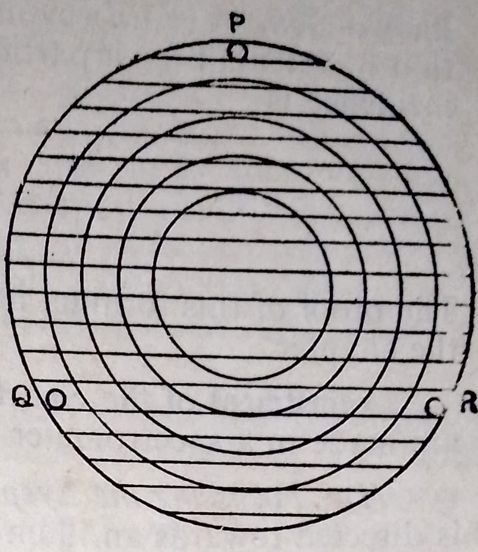
Its essential parts are a collimator, a prism-table and a telescope (fig. 6).



(Fig. 6)

(1) Collimator : It consists of a metallic tube whose axis is fixed and horizontal. At one end of the tube there is a converging lens L_1, and at the other end is a draw tube of metal carrying a slit, the width of which can be varied by means of a screw. The main tube is provided with a rack and pinion arrangement to move the draw tube inside it.

(2) Prism Table : It is a circular table provided with a platform on which the prism (or grating) is placed. The platform can be raised to a suitable height and can be rotated along with the prism table about the axis of the spectrometer. The rotation of the table can be measured by noting the positions of the two verniers V_1 and V_2 on a circular scale. Parallel lines and concentric circles are drawn on the platform. They help in placing the prism in the desired position. The platform is provided with three levelling screws P, Q and R (Fig. 7) while the table is provided with clamping and tangent screws.



(Fig.7)

(3) **Telescope :** It is a horizontal metallic tube having an achromatic convex lens L_2 at one end. At the other end it carries a short coaxial tube containing the cross-wires. Within this tube slides another small tube containing a Ramsden's eyepiece. The distance between the objective and the cross-wires can be altered by a rack and pinion arrangement, while that between the cross-wires and the eyepiece can be altered by hand. The telescope can be rotated about the axis of the spectrometer and is provided with levelling screws, as well as with clamping and tangent screws. Its position can be read on the circular scale (which is attached to it) by means of verniers V_1 and V_2.

Theory : The normal dispersion curve is a graph of the refractive index μ of a prism of a given material against the wavelength of light, λ, in the visible region. It can be fairly-accurately represented by Cauchy's equation

$$\mu = A + \frac{B}{\lambda^2},$$

where A and B are Cauchy's constants which are characteristic of the material of the prism. Therefore, in order to determine A and B, we have to determine μ and λ.

μ for a particular wavelength can be determind by measuring the angle of prism A, and the angle of minimum deviation δ_m produced by the prism in that wavelength. The formula is

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}.$$

The proof of this formula has been given in the beginning of the chapter.

λ can be determined by means of a diffraction grating of known element $(e+d)$ by measuring the angle of diffraction θ for that wavelength for a particular order n. The formula for normal incidence is

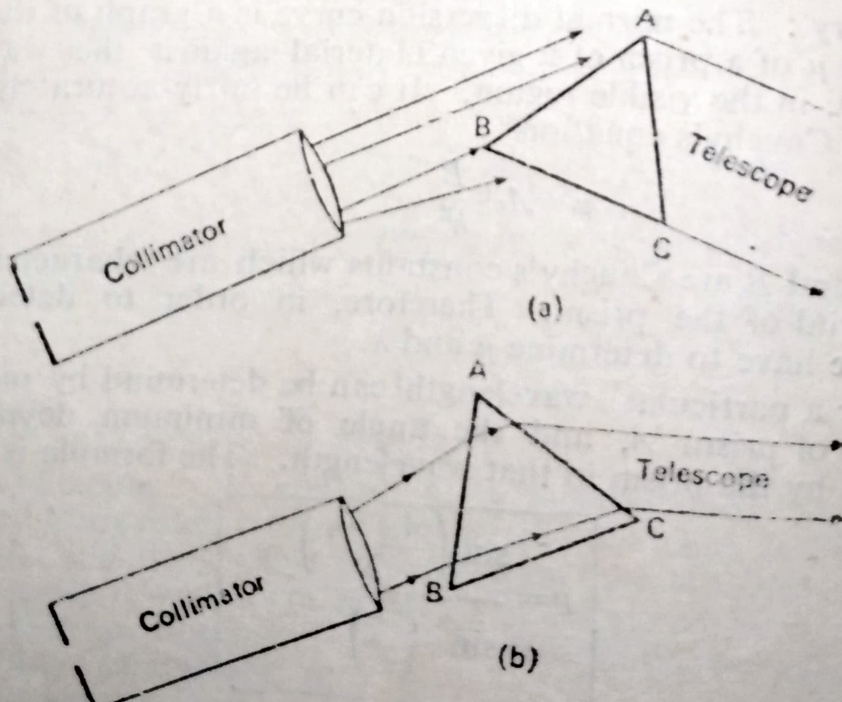
$$\lambda = \frac{(e+d)\sin\theta}{n}.$$

The proof of this formula has also been given in the beginning of the chapter.

Adjustment of the Spectrometer : – The following adjustments are made in a spectrometer before use :

(i) Focussing the Eyepiece on the cross-wires : The telescope is directed towards an illuminated surface (say, a white wall) and the distance between the cross-wires and the eyepiece is altered until the cross-wires are seen as distinct as possible. This focuses the eye-piece on the cross-wir s.

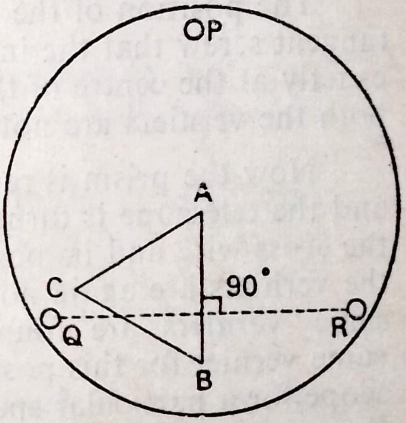
(ii) Focussing of Collimator and Telescope for Parallel Light [Schuster's Method] – The slit of the collimator is made vertical and narrow, and illuminated with sodium light (or mercury light). The prism is placed approximately in the position of minimum deviation. The prism table is now rotated slightly away from this position bringing the refracting edge of the prism towards the telescope (Fig. 8 *a*). The telescope is rotated until the refracted image (or the spectrum) becomes visible in the field of view. The telescope is now focussed on the image as distinctly as possible.



(Fig. 8)

The prism table is now rotated slightly to the other side of the minimum deviation position so that the refracting edge goes towards the collimator (Fig. 8 *b*). Now the collimator is focussed until on looking through the telescope the image (or spectrum) is again as distinct as possible. This process is repeated by alternately focusing the collimator and telescope until the rotations of the prism do not cause the image (or spectrum) to go out of focus. When this is the case, both the collimator and the telescope are in good adjustment, the former for rendering a beam of light parallel, and the later for focussing a parallel beam of light.

(iii) Adjustment of the Prism table—The platform of the prism table is made horizontal with the help of spirit-level and levelling screws *P*, *Q* and *R*. The prism is then placed on the platform such that the refracting edge coincides with the centre, and one of its refracting faces, say *AB* (Fig. 9) is perpendicular to the line joining the two screws *Q* and *R*. The slit is illuminated with sodium (or mercury) light and the prism table is rotated until the light falls equally on both the faces *AB* and *AC* bounding the refracting angle *A*. The telescope is now rotated until the image reflected from the face *AB* becomes visible. If the image does not lie at the centre of the field of view then the levelling screws *Q* and *R* are adjusted until the image becomes central. Next the telescope is rotated to see the image of the slit reflected from the face *AC*. This image is made central in the field of view by adjusting the levelling screw *P* alone. When the images reflected from both the faces of the prism lie in the centre of the field of view, then the prism table is said to be optically levelled.



(Fig, 9)

Procedure : The experiment is performed in two parts : (i) using a prism, (ii) using a grating. In the first part we measure the angle of prism, and the angle of minimum deviations for different spectral lines. In the second part we measure the angle of diffraction for different spectral lines in a particular order and find the grating element.

(i a) Measurement of Angle of Prism (A)—The prism is placed on the prism table as in the adjustment of the prism table *i.e.*, the refracting edge *A* coincides with the centre of the table and the face *AB* remains perpendicular to the line joining the two screws *Q* and *R* (Fig. 9). The table is now rotated so that the light falls equally on both the faces *AB* and *AC* bounding the refracting angle *A*. The prism is then clamped.

The slit of the spectrometer is illuminated with white light (mercury lamp). The telescope is rotated say, to the right hand side and is clamped when the image of the slit formed by reflection at one face of the prism becomes visible in the field of view. Now, by tangent screw the telescope is moved slowly until its vertical cross-wire coincides with the image Both the verniers of the spectrometer are read. The telescope is now rotated to the left hand side and adjusted by means of the tangent screw such that the image of the slit formed by reflection at the other face of the prism coincides with the vertical cross-wire. The verniers of the spectrometer are again read. Half the difference of the two readings of the same vernier gives the refracting angle of the prism *i.e.* A.

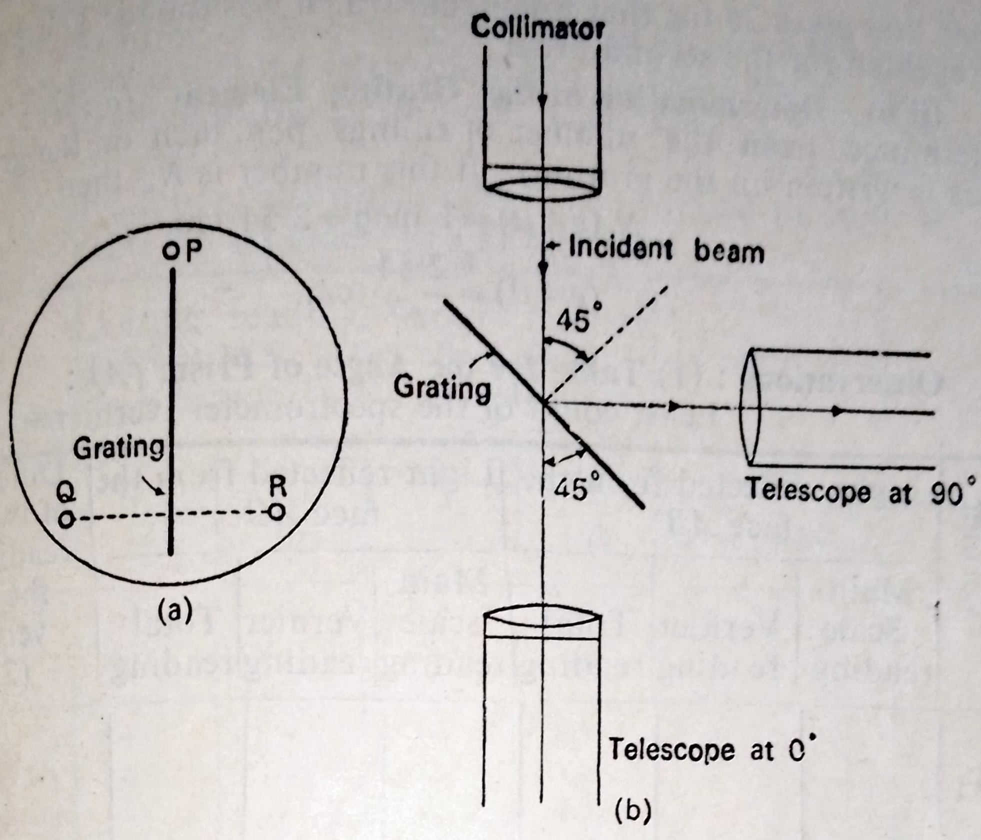
(i b) Measurement of the Angle of Minimum Deviation (δ_m) :— The prism is placed on the prism table such that the main axis of rotation of the instrument passes through the circumcentre of the prism, and the light dispersed through it is received in the telescope *i.e.*, the spectrum appears in the field of view. Now the prism table is slowly rotated in such a direction that the deviation of the rays decreases. The telescope is also rotated so as to keep the spectrum in the field of view. The rotation is continued until the spectrum comes to momentary rest and then returns back. The position of the prism at which the spectrum is just at rest is the position of minimum deviation. The prism table is clamped in this position.

The position of the telescope is so adjusted by means of the tangent screw that the intersection of the cross-wire is, turn by turn, exactly at the centre of the various spectral lines. The readings of both the verniers are noted for each line.

Now the prism is removed without disturbing the prism table and the telescope is turned to receive the direct image of the slit on the cross-wire and its position is again read. The readings of both the verniers are again noted. The difference in the readings of the same verniers are found. The difference in the readings of the same vernier for this position and the previous position of the telescope for a particular spectral line gives the values of δ_m for that line.

(ii a) Measurement of the Angle of Diffraction (θ)—The following adjustments are made with the grating :—

(1) The grating is adjusted on the prism table such that light from the collimator falls 'normally' on it—To do this, the telescope is set in line with the collimator so that the direct image of the slit falls on the intersection of the cross-wires. The position of the telescope is noted. The telescope is turned through 90° from this position and clamped. The axis of the telescope is now perpendicular to that of the collimator. The grating is placed on the prism table such that its ruled surface lies over the centre of the table and is perpendicular to the line joining the levelling screws Q and R (Fig. 10 *a*). The prism table is now rotated till the image of the slit obtained by reflection from the surface of the grating is obtained on the intersection of the cross-wires (Fig. 10 *b*) and then clamped.



(Fig. 10)

Its position is noted on any one vernier. The levelling screws *Q* and *R* are adjusted until the image lies *equally* above and below the intersection of the cross-wires.

In this position the grating surface is at 45° to the incident light. From this position the prism table is rotated through 45° in the proper direction so that the ruled surface of the grating is normal to the incident light and faces the telescope. The prism table is clamped in this position.

(2) **The Rulings of the grating are adjusted parallel to the axis of the spectrometer**—To do this, the diffracted images of the slit (or the spectrum) are observed through the telescope. The levelling screw *P* (Fig. 10 a) is now adjusted until the centres of all the diffracted images lie at the same height in the field of view.

(3) **The rulings are adjusted parallel to the slit**— o do this, the slit is rotated in its own plane until the diffracted images (spectral lines) are as sharp as possible.

Now, to determine θ for the lines whose wavelengths are to be determined, the telescope is rotated to see those lines in the first order on either side of the direct image. The telescope is turned by means of the tangent screw so that these lines fall, turn by turn, on the intersection of the cross-wires, and the readings of both the verniers are taken for each line. The telescope is then turned to the other side of the direct image, and the corresponding readings for the same lines in the first order are again taken. The difference between the two readings of the same vernier for a particular line gives 2θ for that line, from which θ is found. The process is repeated for the second order.

(ii b) Determination of the Grating Element $(e+d)$: It is determined from the number of rulings per inch on the grating (this is written on the grating). If this number is N, then

$$N(e+d)=1 \text{ inch}=2{\cdot}54 \text{ cm.}$$

$$\therefore \quad (e+d)=\frac{2{\cdot}54}{N} \text{ cm.}$$

Observations : (1) Table for the Angle of Prism (A) **:**

Least count of the spectrometer vernier=........

S. No.	Vernier	Light reflected from the face *AB*			Light reflected from the face *AC*			Difference of two readings of the same vernier (2*A*)
		Main Scale reading	Vernier reading	Total reading	Main scale reading	Vernier reading	Total reading	
1 2 3	V_1							
1 2 3	V_2							

Mean 2*A*

(2) Table for the Angle of Minimum Deviation $[\delta_m]$

S. No.	Spectral line (colour)	Vernier reading in Minimum deviation position		Vernier reading in Direct position		Angle of minimum deviation (δ_m)		Mean δ_m $\frac{(a-c)+(b-d)}{2}$
		V_1 (a)	V_2 (b)	V_1 (c)	V_2 (d)	V_1 $(a-c)$	V_2 $(b-d)$	
1	R_1	...	...			...	...	...
2	R_2	...	...			...	...	...
3	Y_1	...	...			...	...	...
4	Y_2	...	...			...	...	...
5	G_1	...	...			...	...	...
6	G_2	...	...			...	...	...
7	G_3	...	...	...	...	...	...	...
8	G_4	...	...			...	...	...
9	B	...	...			...	...	...
10	V_1	...	...			...	...	...
11	V_2	...	...			...	...	...

For the Angle of Diffraction $[\theta]$:—

Reading of the telescope when in line with the collimator=......

Reading of the telescope when rotated through 90°=......

Reading of the prism table when reflected image coincided with cross-wire=......

Reading of the prism table when rotated through 45°=......

Table for 1st order (n=1) Spectrum

S. No.	Spectral line	Vernier reading for R.H.S. Spectrum		Vernier reading for L.H.S. Spectrum		2θ		Mean 2θ	θ
		V_1 (a)	V_2 (b)	V_1 (c)	V_2 (d)	V_1 $(a-c)$	V_2 $(b-d)$	$\frac{(a-c)+(b-d)}{2}$	
1	R_1	...	...	...		...	...	...	...
2	R_2	...	...	...	...	...	...	...	...
3	Y_1	...	...	...	...	...	...	...	...
4	Y_2	...	...	...	...	...	...	...	...
5	G_1	...	...	...	...	...	...	...	...
6	G_2	...	...	...	...	...	...	...	...
7	G_3	...	...	...	...	...	...	...	...
8	G_4		...	...	...	...	...	...	...
9	B	...	...	...	...	...	...	...	...
10	V_1	...	...	...	...	...	...	...	...
11	V_2	...	...	...	...	...	...	...	...

A similar table is drawn for the second order $(n=2)$ spectrum.

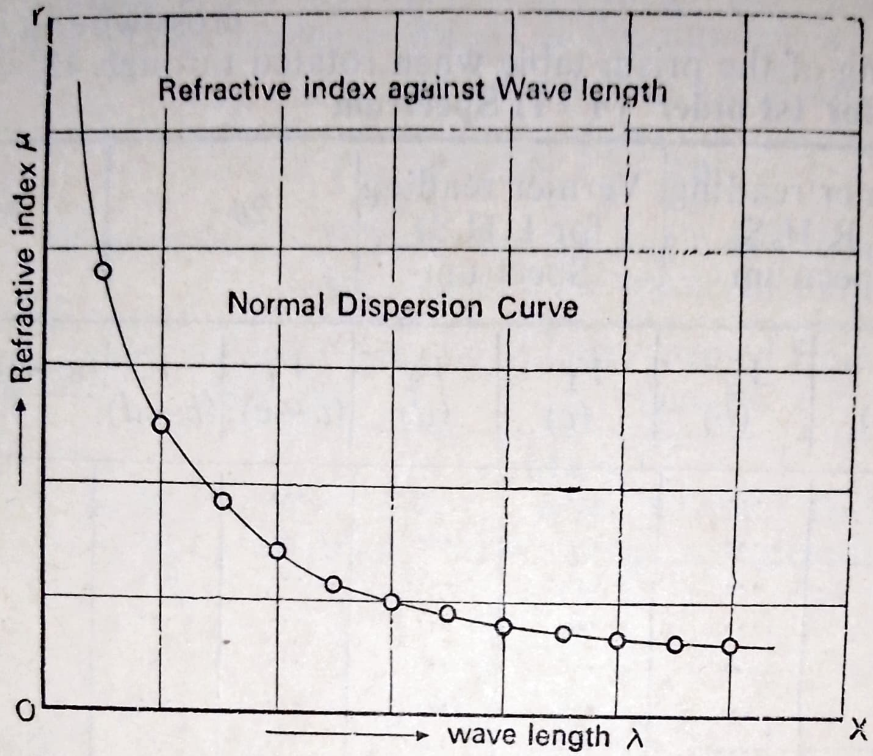
(4) No. of lines per inch on the grating N=......

$$\therefore \text{ grating element } (e+d)=\frac{2{\cdot}54}{N}=\text{......cm.}$$

Calculations : Table for the determination of μ, λ and $1/\lambda^2$

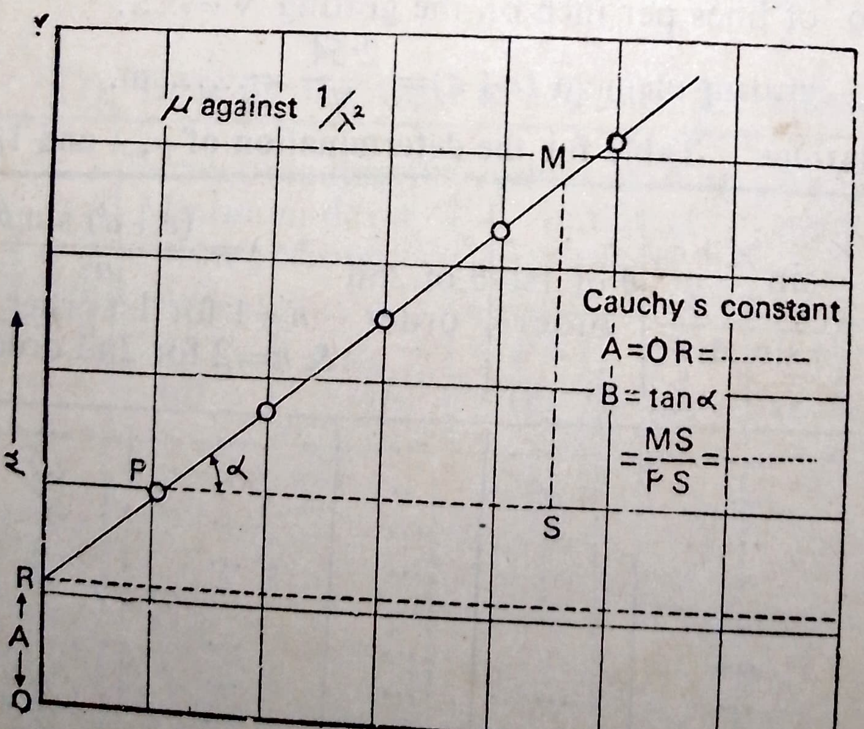
S. No.	Spectral line	$\mu=\dfrac{\sin\frac{A+\delta_m}{2}}{\sin A/2}$	θ of 1st order	θ of 2nd order	$\therefore \lambda=\dfrac{(e+d)\sin\theta}{n}$ $n=1$ for 1st order & $n=2$ for 2nd order		Mean λ	$1/\lambda^2$
1	R_1	...		...	...	...	...	...
2	R_2	...	...	...	...	...	...	...
3	Y_1	...	...	...	...	...	...	...
4	Y_2	...	...	...	...	...	...	...
5	G_1	...	...	...	...	...	...	...
6	G_2	...	...	...	...	...	...	...
7	G_3	...	...	...	...	...	...	...
8	G_4	...	...	...	...	...	...	...
9	B	...	...	...	...	...	...	...
10	V_1	...	...	...	...	...	...	...
11	V_2	...	...	...	...	...	...	...

We plot a graph between the refractive index 'μ' and the corresponding wavelength λ. The curve so obtained (fig. 11) is the normal dispersion curve.



(Fig. 11)

In order to calculate the Cauchy's constants, we plot a curve between 'μ' and '$1/\lambda^2$' as shown in fig. 12. The curve so obtained is a straight line. Its slope gives us the constant '*B*', and its intercept on the μ axis gives the constant '*A*'.



(Fig. 12)

From fig. 12 we have

$$A = OR = \ldots\ldots$$

$$B = \frac{MS}{PS} = \ldots\ldots \text{cm}^2.$$

Results : (1) The curve for normal dispersion is represented on the graph plotted between μ and λ.

(2) The values of Cauchy's constants obtained from graph [μ, $1/\lambda^2$] :

$$A = \ldots\ldots$$

$$B = \ldots\ldots \text{cm}^2.$$

Precautions :

(1) The adjustment of the spectrometer should be done systematically. The collimator and the telescope should be adjusted for parallel rays and the prism table should be levelled mechanically as well as optically.

(2) The slit should be vertical and narrow and uniformly illuminated along its length.

(3) The ruled surface of the grating should face the telescope and should be normal to the incident light.

(4) The readings of both the verniers should be noted and the difference of the readings should be taken for the same vernier.

(5) The surface of the prism or the grating should not be touched with fingers.

Criticism—The Cauchy's equation does not exactly agree with the experimental normal dispersion curve. It is in fact an approximation to 'Sellmeier's equation' given by

$$\mu^2 = 1 + \frac{a_0 \lambda^2}{\lambda^2 - \lambda_0^2},$$

where a_0 and λ_0 are two constants, the latter being the wavelength in vacuum. This equation gives a more accurate representation of μ in regions far from absorption bands than does the Cauchy equation with same number of constants.

If the Cauchy's formula is taken to three or four terms, it represents the experimental facts more accurately but proves unsatisfactory when applied to the region of infra-red. It also fails to account for the anomalous dispersion.