Note down comments, sources of errors and precautions on the basis of determined values.

EXPERIMENT 7

Object. *A study of prismatic spectrum by using Hartmann relation : Determination of constants occurring in the relation.*

The basis to use Hartmann dispersion formula in studying prismatic spectrum is as given below and either visual of photographic measurements are made accordingly.

THE HARTMANN DISPERSION FORMULAE

The dispersion of light is fairly accurately given by Cauchy's dispersion relation in visible region. It can be slightly made more accurate to fit in results by adding a third term, then it assumes the following form

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} \quad \text{...(7.1)}$$

where A, B, C are constants of the dispersing medium. The values of constants are different for normal and anomalous dispersions. Hartmann has given another empirical relation for dispersion which applies for a wider range of spectrum and gives better results. The relation is written as

$$\mu = \mu_0 + \frac{C}{\lambda - \lambda_0} \quad \text{...(7.2)}$$

where μ_0, λ_0 and C are constants. This can be written as

$$\boxed{\lambda = \lambda_0 + \frac{C}{\mu - \mu_0}} \quad \text{...(7.3)}$$

This relation can be used to find wavelength (λ) corresponding to determined refractive index (μ). The constants λ_0, μ_0 and C can be evaluated by determining μ experimentally for there standard wavelengths, say λ_1, λ_2, λ_3. In order to facilitate the determination, the terms μ_0 and μ are replaced by linear distances (d_0) and (d) of spectral lines from an orbitrary zero. The distances of spectral lines should be measured fairly accurately with a vernier microscope having least count of the order of wavelength (10^{-5} cm). The same applies to relation (7.3) where μ should be measurable to an accuracy of the order of 10^{-5} for correct evaluation of λ. Since distance measurement is more convenient and accurate and the experimental arrangement also becomes simpler, the relation (7.3) is written as follows in its most useful form.

$$\boxed{\lambda = \lambda_0 + \frac{C}{d_0 - d}} \quad \text{...(7.4)}$$

The position of the spectral lines are always measured in the direction of increasing wavelength. The constant (λ_0) is a constant of the instrument while (d_0) depends on setting of scale and (C) depends upon the wavelength range and region of the spectrum in use. Writing d_1, d_2, d_3 corresponding to λ_1, λ_2, λ_3 respectively we get

$$\lambda_1 = \lambda_0 + \frac{C}{d_0 - d_1} \qquad ...(i)$$

$$\lambda_2 = \lambda_0 + \frac{C}{d_0 - d_2} \qquad ...(ii)$$

$$\lambda_3 = \lambda_0 + \frac{C}{d_0 - d_3} \qquad ...(iii)$$

From [(ii) - (i)] and [(iii)-(i)] we get

$$\frac{d_2 - d_1}{\lambda_2 - \lambda_1} = \frac{(d_0 - d_2)(d_0 - d_1)}{C} = A \qquad ...(iv)$$

$$\frac{d_3 - d_1}{\lambda_3 - \lambda_1} = \frac{(d_0 - d_3)(d_0 - d_1)}{C} = B. \qquad ...(v)$$

From [(iv)-(v)] we get

$$C = \frac{(d_0 - d_1)(d_3 - d_2)}{A - B} = \frac{(d_0 - d_1)(d_3 - d_2)}{(d_2 - d_1)/(\lambda_2 - \lambda_1) - (d_3 - d_1)/(\lambda_3 - \lambda_1)} \qquad ...(vi)$$

Substituting the value of C in relation (i) we get

$$\lambda_0 = \lambda_1 - \frac{d_3 - d_2}{A - B}\, . \qquad ...(vii)$$

From relation (ii) we get

$$\frac{1}{\lambda_2 - \lambda_0} = \frac{d_0 - d_2}{C}$$

Substituting the above value in eq. (iv) we get

$$d_0 = A\,(\lambda_2 - \lambda_0) + d_1. \qquad ...(viii)$$

The sequence of calculation in using Hartmann relation becomes as given below

$A = (d_2 - d_1)/(\lambda_2 - \lambda_1)$

$B = (d_3 - d_1)/(\lambda_3 - \lambda_1)$

$$\lambda_0 = \lambda_1 - [(d_3 - d_2)/(A - B)] \qquad ...(ix)$$
$$d_0 = A(\lambda_2 - \lambda_0) + d_1.$$
$$C = (d_0 - d_1)(d_3 - d_2)/(A - B)$$

The discussion of Hartmann dispersion formulae shows that relation (7.3) is used with visual observations taken by a spectrometer and prism while relation (7.4) is used with a photographed spectrum. However photographing the spectrum and making measurements with it has been given as Expt. (8) with constant deviation spectrograph while the method is discussed as an auxiliary Expt. (A.7). We discuss the use of relation (7.3) below.

Experimental set up. Select a good quality spectrometer, sources of light (Hg, Na, He) and usual accessories used to set up an experiment to make spectroscopic measurements. Refer to Expt. (A.6) to adjust and set up a spectrometer to make measurements with it. Also refer to table (6.1) given in Expt. (6) to select three standard wavelengths λ_1, λ_2, λ_3. Let us select three easily distinguishable spectral lines of mercury spectrum say λ_1 = 5461 Å green, λ_2 = 4358 Å blue and λ_3 = 4037 Å violet. The refractive indices corresponding to these wavelengths are experimentally measured as μ_1, μ_2, μ_3 respectively by usual relation :

$$\mu = \sin[(A + D)/2]/\sin(A/2) \qquad ...(7.5)$$

where A is the angle of the prism used to disperse the spectrum and D is the anlge of minimum deviation for the spectral line under observation. Knowing λ_1, λ_2, λ_3 and determining corresponidng μ_1, μ_2, μ_3 calculate the values of constants λ_0, μ_0 and C. However unknown wavelength is then determined by measuring corresponding μ by the same set of spectrometer and prism and substituting λ_0, μ_0, C in relation (7.3). The experimental procedure in brief is given below :

Experimental Procedure. Set up spectrometer and prism on it to find the angle of prism A as per description in Expt. (A.6). Record related observations systematically and determine the value of A. However the usual triangular prisms have A = 60° which can also be determined by geometrical method. Next set the prism and form Hg-spectrum to record angles of minimum deviation for constituent wavelengths as per description in Expt. (A.6). Measure the values of D's for all spectral lines and record observations in tabular form. Sort out the values of D for λ_1 = 5461 Å green, λ_2 = 4358 Å blue and λ_3 = 4037 Å violet. Calculate the values of μ_1, μ_2, μ_3 corresponding to D values for λ_1, λ_2, λ_2 by using relation (7.5) ; tabulate λ_1, λ_2, λ_3 and μ_1, μ_2, μ_3 and form three equations by substituting