Study of all Subgroups of the Symmetric Group S_6

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Abstract: In this paper, we aimed at determining all subgroups of the Symmetric group S_6 up to Automorphism class using Sylows theorem and Lagranges theorem. This is achieved by finding all subgroups of order m for which $\frac{m}{O(S_6)}$ and are subsets of S_6. Further, the Symmetric group S6 is centerless and every automorphism of it is inner. Also, every natural homomorphism to the automorphism group is an isomorphism.

Keywords: Symmetric group, Conjugacy class, Isomorphism, Automorphism, Complete group

1.**Introduction**

In mathematics, the notion of permutation is used with several slightly different meanings, all related to the act of permuting (rearranging in an ordered fashion) objects or values. Informally, a permutation of a set of values is an arrangement of those values into a particular order. Thus there are six permutations of the set 1,2,3, namely, (1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), and (3,2,1). In algebra and particularly in group theory, a permutation of a set S is defined as a bijection from S to itself . To such a map f is associated with the rearrangement of S in which each element s takes the place of its image f(s). Given any non empty set S, define A(S) to be the set of all bijections mapping of the set S onto itself. The set A(S) is a group with respect to composition of function. If the set S is finite with n elements, then the group A(S) is denoted by Sn. The order of Sn is n! And will be called Symmetric group. Any subset of S_n which is itself a group is called a subgroup of Sn. There are many references on subgroups of S_2, S_3, S_4 and S_5 ([2], [7], [8] and [10]). Our aim in this paper is to critically examine all subgroups of S6 up to automorphism class and their conjugacy class size. The set of all symmetry operations on all objects in the set S, can be modeled as a group action g : G $\times$ S $\to$ S, where the image of g in G and x in S is written as gx. If, for some g, gx = y then x and y are said to be symmetrical to each other. For each object x, operations g for which gx = x is the symmetry group of the object, a subgroup of G. If the symmetry group of x is the trivial group then x is said to be asymmetric, otherwise symmetric.

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II. **Preliminary**

Definition 1: The symmetric group S_6 is defined in the following equivalent ways: It is the group of all permutations on a set of five elements, i.e., it is the Symmetric group of degree five. In particular, it is a symmetric group of prime degree and symmetric group of prime power degree. With this interpretation, it is denoted S_6. Equivalently, it is the projective general linear group of degree two over the field of five elements, [5].

Definition 2: Let G be a group and let N be a proper normal subgroup of G. Then N is called maximal subgroup of G if there does not exists any proper normal subgroup M of G such that N $\leq$ M $\leq$ G [12].

Definition 3: A homomorphism ϕ: G $\rightarrow$ K from a group G to a group K is a function with the property that ϕ $(g_1 g_2) = \phi(g_1)$ ϕ (g_2) for all g_1, g_2 $\in$ G, [9].

Definition 4: An isomorphism ϕ: G $\rightarrow$ K between two groups G and K is a homomorphism that is also a bijection mapping G onto K. Two groups G and K are isomorphic if there exists an isomorphism mapping G onto K, written as G $\cong$K. While an automorphism is an isomorphism mapping a group onto itself [9].

Definition 5: A group is said to be complete if it satisfies the following equivalent conditions
1. It is centerless and every automorphism of it is inner.
2. The natural homomorphism to the automorphism group, that sends each element to the conjugation via that element is an isomorphism.
3. Whenever it is embedded as a normal subgroup inside a bigger group, it is actually a direct factor inside that bigger group.
Equivalently; A group G is said to be complete if it satisfies the following equivalent conditions
: 1. The center of G i.e. Z(G) is trivial and Inn(G) = Aut(G) (i.e. every automorphism of G is inner),
2.The natural homomorphism G $\rightarrow$Aut(G) given by g $\rightarrow$Cg (where Cg= x$\rightarrow$gxg^-1) is an isomorphism,
3. For any embedding of G as a normal subgroup of some group K, G is a direct factor of K [6].

Definition 6: A partial ordered on a nonempty set P is a binary relation $\leq$ on P that is reflexive, anti symmetric and transitive. The pair $\langle$ P, $\leq$ $\rangle$ is called a partially ordered set or poset. Poset $\langle$ P, $\leq$ $\rangle$is totally ordered if every x, y $\in$ P are comparable, that is x $\leq$ y or y $\leq$ x. A nonempty subset

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S of P is a chain in P if S is totally ordered by $\leq$ [11].

Theorem 1: (Lagranges Theorem) If G is a finite group and H is a subgroup of G, then order of H is a divisor of order of G [7].

Theorem 2: If G is a finite group and x∈ G, then order of x is a divisor of order of G [7].

Theorem 3: (Cauchys Theorem) Let G be a finite group and let p be a prime number that divides the order of G. Then G contains an element of order p [3].

Theorem 4: (The First Sylow Theorem) Let G be a finite group and let $|G|= p^n m$ where n¿1 , p is a prime number and (p, m) = 1. Then G contains a subgroup of order pk for each k where 1 ≤ k ≤ n [8].

Theorem 5: (Second Sylow Theorem) Let G be a finite group, and let p be a prime number dividing the order of G. Then all Sylow p-subgroups of G are conjugate, and any p-subgroup of G is contained in some Sylow p-subgroup of G. Moreover the number of Sylow p-subgroups in G divides the order of G and is congruent to 1 modulo p [8].

Theorem 6: (The Third Sylow Theorem) Let G be a finite group and let | G | = $p^n m$ where n ≥1, p is a prime number and (p, m) = 1. Then the number of Sylow psubgroup is of the form (1+ kp), where k is a non-negative integer, and (1 + kp) divides the order of G [8].

Theorem 7: There is a unique Sylow psubgroup of the finite group G if and only if it is normal [2].

Theorem 8: Let G be a group of order pq, where p and q are distinct primes and p < q. Then G has only one subgroup of order q. This subgroup of order q is normal in G [2].

Definition 9: A subgroup N of G is said to be a normal subgroup of G if for every g∈ G and n ∈ N, $gng^{-}1$ ∈ N [7].

Definition 10: A non-trivial group G is said to be simple if the only normal subgroups of G are the whole of G and the trivial subgroup { e } whose only element is the identity element e of G [3].

III. The Symmetric Group S6

The Symmetric group S_6 is the group of permutations of the set S = {1, 2, 3, 4, 5, 6 }, i.e. if then the set of all bijections f :S →S defined by α(a)= a ;i, j <6 is called the Symmetric group S_6.

The collection of all such permutations gives rise to a group of order 720 as follows:

3.1 Identity permutation = I
3.2 Number of transposition:
(12),(13),(14),(15),(16)(23),(24),(25),(26), (34),(35),(36),(45),(46),(56).
3.3 Number of one 3-cycle:

(123), (132), (124), (142), (125), (152),(126), (162), (345), (354)(134), (143), (135), (153), (136), (163), (145), (154), (346), (364),(146), (164), (156), (165), (234), (243), (235), (253), (356), (465), (236), (263), (245), (254), (246), (264), (265), (456), (256), (365).

3.4 Number of two transposition:

(24)(65),(24)(35),(12)(34),(12)(35),(12)(36),(12)(45),(12)(46),(34)(56), (13)(45),(13)(46),(13)(65),(32)(65),(13)(25), (13)(26),(35)(46),(62)(54), (14)(23),(14)(25),(14)(26),(14)(35),(14)(36),(36)(45),(62)(53),(14)(56), (15)(23),(15)(24), (15)(62),(15)(43),(13)(24),(25)(36),(15)(46),(15)(36), (16)(24),(16)(25),(12)(56),(25)(46),(62)(34),(16)(23),(16)(34), (16)(53), (23)(45),(23)(46),(42)(63),(25)(34), (36)(14)

3.5 Number of one 4-cycle:
(1243),(1234),(1235),(1253),(1236),(1263),(1625),(1652),(1563),(1546),(1564), (1245),(1254),(1246),(1264),(1256), (1342),(1324),(1325),(1352),(1362),(1345), (1354),(1346)(1356),(1543),(1534),(1562),(1526),(1524),(1542),(1423), (1432), (1523),(3245),(5136),(2365),(6354),(3645),(1452),(1462),(1426),(1435),(1453), (1456),(1465),(1463),(1532), (1623),(1632),(1624),(1642),(1645),(1654),(3254), (1634),(1643),(1653),(2654),(5142),(6143),(2536),(2563),(5163), (6132),(4256), (2345),(2354),(2346),(2364),(2356),(3246),(3264),(3265),(2435),(2436),(4253), (5264),(5246),(5263), (6254), (6245),(4263),(3456),(4356),(4365),(3465),(1536), (1265), (1364)
3.6 Number of one 5-cycle:
(12345),(12354),(13254),(13245),(14352),(14325),(15234),(15243),(13542), (14523),(16243),(13642),(14623),(15263),(15324),(15342),(15423), (15432),(13425),(13452),(13524),(12453),(12435),(12534),(12543), (14235),(14253),(14532), (12346),(12364),(13264),(13246),(14362), (14326),(16234),(16324),(16342),(16423),(16432),(13426),(13462), (13624), (12463),(12436),(12634),(12643),(14236),(14263),(14632), (12365),(12356),(13256),(13265),(16352),(16325), (15236), (15326), (15362),(15623),(15632),(13625),(13652),(13526),(13562),(16523), (15246),(16542),(14526),(15643),(13546), (12653),

(12635),(12536),(12563),(16235),(16253),(16532),(12645),(12654),
(16254),(16245),(14652),(14625),(15264), (15624),(15642),(15426)
,(,15462),(16425),(16452),(16524),(12456),(12465),(12564),(12546),
(14265),(14256),(14562),(16345),(16354),(13654),(13645),(14356),
(14365),(15634),(15364),(15346),(15463),(15436),(13465),(13456),
(13564), (16453),(16435),(16534),(16543),(14635),(14653),(14536),
(14563),(65243),(63542),(64523),(62345),(62354),(63254), (63245),
(64352),(64325),(65234),(65324),(65342),(65423),(65432),(63425),
(63452),(63524),(62453),(62435),(62534), (62543),(64235),(64253),
(64532)

3.7 Number of two triple cycle:
(123)(456),(132)(456),(124)(356),(142)(356),(125)(346),(152)(346),
(153)(246),(135)(246),(164)(253),(142)(365), (126)(345)'(162)(345),
(134)(256),(143)(256),(136)(245),(163)(245),(145)(236),(154)(236),
(146)(253),(156)(243), (165)(243),(123)(465),(132)(465),(124)(365),
(125)(364),(152)(364),(126)(354),(162)(354),(134)(265),(143)(265),
(154)(263),(135)(264), (153)(264), (136)(254), (163)(254),(145)(263),
(146)(235), (164)(235), (156)(234), (165)(234)

3.8 Number of Triple Transposition :
(12)(34)(56),(13)(24)(56),(14)(23)(56),(24)(16)(35),(34)(16)(25),
(15)(24)(36),(35)(26)(14),(45)(12)(36),(16)(23)(45), (26)(13)(45),
(46)(12)(35),(23)(45)(16),(15)(23)(46),(25)(13)(46),(26)(15)(34)

3.9 Number of one 6-cycle:
(152436),(152346),(135246),(145326),(123456),(123546),(132546),
(132456),(143526),(143256),(153246),(153426), (154236),(154326),
(134256),(134526),(124536),(124356),(125346),(125436),(142356),
(142536),(123465),(123645), (132645),(132465),(143625),(143265),
(162345),(136245),(146325),(152364),(135264),(135426),(163245),
(163425), (164235),(164325),(134265),(134625),(124635),(124365),
(126345),(126435),(142365),(142635),(123654),(123564), (132564),
(132654),(163524),(163254),(153264),(153624),(156234),(156324),
(136254),(136524),(135624)(152634), (146235),(136425),(162435),
(145236),(126534),(126354),(125364),(125634),(162354),(162534),
(165324),(152463), (165243),(145623),(156342),(135642),(126453),
(126543),(162543),(146523),(146253),(152643),(156243),(156423),
(154263),(154623),(164253),(164523),(124563),(124653),(125643),
(125463),(142653),(142563),(163452),(163542), (136542),(136452),
(143562),(143652),(153642),(153462),(154632),(154362),(134652),

(134562),(164532),(164352), (165342),(165432), (146352),(146532),
(145632),(135462),(156432),(145263),(165423),(624531),(145362),
(652341)

3.10 . Number of one 2- cycle and one 3-cycle:
(12)(345),(12)(354),(13)(245),(13)(254),(14)(123),(14)(132),(15)(243),(25)(134),
(35)(624),(35)(642),(16)(243), (26)(134),(45)(623),(15)(234),(23)(145),(23)(154),
(24)(135),(24)(153),(25)(134),(34)(125),(34)(152),(35)(124), (35)(142),(45)(123),
(45)(132),(12)(346),(12)(364),(13)(246),(13)(264),(14)(123),(14)(132),(16)(234),
(23)(146), (23)(164),(24)(136),(24)(163),(26)(134),(34)(126),(34)(162),(36)(124),
(36)(142),(46)(123),(46)(132),(12)(365), (12)(356),(13)(265),(13)(256),(16)(123),
(16)(132),(15)(263),(25)(136),(45)(632),(15)(246),(25)(164),(34)(625), (15)(236),
(23)(165),(23)(156),(26)(135),(26)(153),(25)(163),(36)(125),(36)(152),(35)(126),
(35)(162),(65)(123), (65)(132),(12)(645),(12)(654),(16)(245),(16)(254),(14)(126),
(14)(162),(15)(264),(26)(145),(26)(154),(24)(165), (24)(156),(25)(164),(64)(125),
(64)(152),(65)(124),(65)(142),(45)(126),(45)(162),(16)(345),(16)(354),(13)(645),
(13)(654),(14)(163),(14)(136),(15)(643),(65)(134),(62)(345),(34)(652),(25)(634),
(25)(643),(15)(634),(63)(145), (63)(154),(64)(135),(64)(153),(65)(134),(34)(165),
(34)(156),(35)(164),(35)(146),(45)(163),(45)(136),(62)(354), (63)(245),(63)(254),
(64)(123),(64)(632),(65)(243),(65)(234),(23)(645), (23)(654),(24)(635), (24)(653)

3.11 Number of one 4- cycle and one 2-cycle:
(1243)(56),(1234)(56),(1235)(46),(1253)(46),(1236)(45),(1263)(45),(1564)(23),
(1265)(34),(1345)(26),(1534)(26), (1625)(34),(1652)(34),(1536)(24),(1563)(24),
(1546)(23),(1245)(36),(1254)(36),(1246)(35),(1264)(35),(1256)(34), (1342)(56),
(1324)(56),(1325)(46),(1352)(46),(1362)(45),(1354)(26),(1346)(25),(1364)(25),
(1356)(24),(1543)(26), (1562)(34),(1526)(34),(1524)(36),(1542)(36),(1423)(56),
(1432)(56),(1456)(23),(1632)(45),(3245)(16),(6143)(25), (1452)(36),(1462)(35),
(1426)(35),(1435)(26),(1453)(26),(1465)(23),(1463)(25),(1523)(46),(1532)(46),
(1623)(45), (1624)(35),(1642)(35),(1645)(23),(1654)(23),(3254)(16),(1634)(25),
(1643)(25),(1653)(24),(2654)(13),(5142)(36), (2536)(14),(2563)(14),(5136)(24),
(5163)(24),(6132)(45),(4256)(13),(2365)(14),(4253)(16),(4263)(15),(3645)(12),
(2345)(16),(2354)(16),(2346)(15),(2364)(15),(2356)(14),(3246)(15),(3264)(15),
(3265)(14),(2435)(16),(2436)(15), (5264)(13),(5246)(13),(5263)(14),(6254)(13),
(6245)(13),(3456)(12),(4356)(12),(4365)(12), (3465)(12), (6354)(12)

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IV. Main Results

According to the Lagranges theorem, the order of any non-trivial subgroup of S_6 divides the order of S6. Therefore we shall determine all subgroups of S_6 and their isomorphism class. Obviously, the only subgroup of S_6 of order 1 is the trivial subgroup G1 = I.

(1) There are two trivial sub groups of a group S_6 which are:

$$S_6, \text{I}$$

(2) The sub groups of S_6 which has two elements are:

[I,(12),I,(12)(34),I,(12)(35),I,(12)(34)(56),I,(12)(46), I,(13),I,(12)(36),I,(12)(45), I,(16)) ,I,(13)(25),I,(13)(45),I,(13)(26),I,(13)(24)(56),I,(13)(46) , I,(15) , I,(14)(35) ,I,(13)(24),I,(23),I,(23)(15)(46),I,(23)(54),I,(23)(14),I,(14)(23)(56), I,(14)(56) ,I,(14)(36) ,I,(14)(25),I,(14)(26),I,(34)(16)(25),I,(34)(56), I,(35), I,(12)(35)(46), I,(24),I,(24)(16)(35),I,(35)(46),I,(13)(25)(46), I,(25)(36),I,(45), I,(16)(25), I,(15)(24)(36) ,I,(26),I,(12)(36)(45),I,(36)(45),I,(25),I,(23)(15), I,(15)(24) ,I,(15)(26) ,I,(15)(34), I,(26)(35) , I,(16)(23)(45) ,I,(23)(16),I,(16)(34), I,(16)(35),I,(13)(26)(45), I,(26)(54),(I,(14)(36)(25),I,(15)(23)(46),I,(46), I,(23)(14)(56), I,(56) ,I,(26)(34) ,I,(25)(46) ,I,(15)(46),I,(24)(36),I,(24)(35), I,(15)(36),I,(12)(56), I,(25)(34), I,(23)(46),I,(24)(56) ,I,(34),I,(16)(24),I,(13)(65), I,(23)(56) ,I,(36)].

(3) The sub groups of S_6 which has three elements are;
I,(123),(132),I,(124),(142),I,(134),(143), I,(234),(243),I,(235),(253),I,(345),(354), I,(245),(254),I,(125),(152),I,(145),(154), I,(135),(153),I,(126),(162), I,(146),(164), I,(136),(163), I,(236),(263),I,(156),(165),I,(246),(264), I,(256),(265),I,(346),(364), I,(356),(365),I,(456),(465).

(4) The sub groups of S_6 which has four elements are
I,(12),(34),(12)(34) ,I,(12),(35),(12)(35),I,(12),(36),(12)(36),I,(12),(45) (12)(45), I,(13),(25), (13)(25),I,(13),(45),(13)(45),I,(13),(26),(13)(26),I,(13),(46),(13)(46) , I,(14),(35),(14)(35),I,(13),(24),(13)(24),I,(23),(54),(23)(54),I,(23),(14),(23)(14), I,(14),(56),(14)(56),I,(14),(36),(14)(36),I,(14),(25),(14)(25) ,I,(14),(26),(14)(26) , I,(34),(56) ,(34)(56),I,(35),(46),(35)(46), I,(25),(36),(25)(36),I,(16),(25),(16)(25), I,(36),(45), (36)(45),I,(23),(15) ,(23)(15), I,(15),(24),(15)(24),I,(15),(26) ,(15)(26) ,I,(15),(34),(15)(34),I,(26),(35),(26)(35),I,(23),(16),(23)(16),I,(16),(34),(16)(34), I,(16),(35),(16)(35),I,(26),(54),(26)(54),I,(26),(34),(26)(34),I,(25),(46),(25)(46), I, (15),(46),(15)(46),I,(24),(36), (24)(36),I,(24),(35),(24)(35),I,(15),(36),(15)(36), I,(12),(56), (12)(56),I,(25),(34), (25)(34), I,(23),(46),(23)(46),I,(24),(56),(24)(56), I,(16),(24), (16)(24),I,(13),(65) ,(13)(65),I,(23)(56) (23)(56)

(5) The sub groups of a group S_6 with order five are:
I,(13),(24),(56),(13)(24)(56),I,(23),(15),(46),(23)(15)(46),I,(34),(16),(25), (34)(16)(25), I,(12),(35),(46),(12)(35)(46),I,(24),(16),(35),(24)(16)(35) ,I,(13),(25),(46),(13)(25)(46), I,(14),(23),(56),(14)(23)(56),I,(15),(23),(46),

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(15)(23)(46)I,(15),(24),(36),(15)(24)(36), I,(14),(36),(25),(14)(36)(25), I,(12),(36),(45),(12)(36)(45),I,(23),(14),(56),(23)(14)(56), I,(16),(23),(45) ,(16)(23)(45),I,(13),(26),(45),(13)(26)(45)

(6) The sub groups of S_6 which has six elements are;
I,(23),(24),(34),(234),(243),I,(25),(35),(23),(235)(253),
I,(25),(45),(24),(254),(245), I,(12),(14),(24),(124),(142),
I,(15),(25),(12),(125),(152), I,(13),(14),(34),(134),(143),
I,(15),(35),(13),(135),(153),
I,(14),(15),(45),(145),(154), I,(12),(13),(23),(123),(132),
I,(15),(25),(12),(125),(152), I,(13),(15),(35)(135),(153),
I,(16),(26),(12),(126),(162), I,(13),(36),(16),(136),(163),
I,(14),(16),(46),(146),(164), I,(16),(56),(15),(156),(165),
I,(23),(26),(36),(236),(263), I,(24),(26),(46),(246),(264),
I,(25),(26),(56),(256),(265), I,(34),(35),(45),(354),(345),
I,(36),(46),(34),(346),(364), I,(35),(36),(56),(356),(365),
I,(45),(46),(56),(456),(465)

(7) The sub groups of S_6 which has eight elements are;
I,(1234),(13)(24),(1432),(13),(24),(12)(34),(14)(23),
I,(2345),(24)(35),(2543),(24),(35),(23)(45),(25)(34),
I,(1245),(14)(25),(1542),(14),(25),(12)(45),(15)(24),
I,(1345),(14)(35),(1543),(14),(35),(13)(45),(15)(34),
I,(1235),(13)(25),(1532),(13),(25),(12)(35),(15)(23),
I,(1236),(13)(26)(1632),(13),(26),(12)(36),(16)(23),
I,(1246),(16)(24),(1462),(16),(24),(12)(64),(14)(26),
I,(1256),(15)(26),(1652),(15),(26),(12)(56),(16)(25),
I,(1634),(13)(64),(1436),(13),(64),(16)(34),(14)(63),
I,(1635),(13)(65),(1536),(13),(65),(16)(35),(15)(63),
I,(1564),(16)(54),(1465),(16),(54),(15)(64),(14)(56),
I,(2346),(63)(24),(6432),(63),(24),(62)(34),(64)(23),
I,(6235),(63)(25),(6532),(13),(26),(62)(35),(65)(23),
I,(6254),(63)(24),(6432),(63),(24),(62)(34),(64)(23).

(8) The sub groups of S_6 which has twelve elements are;
I,(1234),(13)(24),(1432),(13),(24),(12)(34),(14)(23),(12),(34),(14),(23).
I,(2345),(24)(35),(2543),(24),(35),(23)(45),(25)(34),(23),(45),(25),(34).
I,(1245),(14)(25),(1542),(14),(25),(12)(45),(15)(24), (12),(45),(15),(24).
I,(1345),(14)(35),(1543),(14),(35),(13)(45), (15)(34),(13),(45),(15),(34).
I,(1235),(13)(25),(1532),(13),(25) ,(12)(35),(15)(23), (12),(35),(15),(23) .
I,(1236),(13)(26),(1632) ,(13),(26),(12)(36),(16)(23),(12),(36),(16),(23).

I,(1246),(16)(24) ,(1462),(16),(24),(12)(64),(14)(26),(12),(64),(14),(26).
I,(1256),(15)(26),(1652),(15),(26),(12)(56), (16)(25)(12),(56),(16),(25).
I,(1634),(13)(64),(1436),(13),(64),(16)(34),(14)(63)(16),(34),(14),(63).
I,(1635),(13)(65),(1536),(13),(65),(16)(35),(15)(63),(16),(35),(15),(63).
I,(1564),(16)(54),(1465),(16),(54),(15)(64),(14)(56),(15),(64) ,(14),(56).
I,(2346),(63)(24),(6432),(63),(24),(62)(34),(64)(23),(62),(34),(64),(23).
I,(6235),(63)(25),(6532),(13),(26),(62)(35),(65)(23) ,(62),(35),(65),(23).
I,(6234),(63)(24),(6432),(63),(24),(62)(34),(64)(23) ,(62),(34),(64),(23).

(9) The sub groups of S_6 which has fifteen elements are;
I,(1234),(13)(24),(1432),(13),(24),(12)(34),(14)(23),(12),
(34),(14),(23)(13),(24),(1324).
I,(2345),(24)(35),(2543),(24),(35),(23)(45),(25)(34),(23),(45),(25),(34),
(24),(35), (2435).
I,(1245),(14)(25),(1542) ,(14),(25),(12)(45),(15)(24),(12),(45),(15),
(24,(14),(25),(1425)).
I,(1345),(14)(35),(1543),(14),(35),(13)(45),(15)(34),(13),(45),(15),
(34),(14),(35),(1435).
I,(1235),(13)(25),(1532),(13),(25),(12)(35),(15)(23), (12),(35),
(15),(23), (13),(25), (1325).
I,(1236),(13)(26) ,(1632),(13),(26),(12)(36),(16)(23),(12),(36),
(16),(23), (13),(26) ,(1326) .
I,(1246),(16)(24),(1462),(16),(24),(12)(64),(14)(26),(12),(64) ,
(14),(26), (16),(24) (1624).
I,(1256),(15)(26) ,(1652),(15), (26), (12)(56), (16)(25) (12), (56),(16),
(25) ,(15),(26),(1526).
I,(1634),(13)(64),(1436),(13),(64),(16)(34),(14)(63)(16),(34),
(14),(63),(13),(64),(1364).
I,(1635),(13)(65),(1536),(13),(65),(16)(35),(15)(63),(16),(35),(15),
(63), (13),(65), (1365).
I,(1564),(16)(54),(1465),(16),(54) ,(15)(64),(14)(56),(15),
(64),(14),(56),(16),(54),(1654).
I,(2346),(63) (24),(6432),(63),(24),(62)(34),(64)(23),(62),(34),
(64),(23),(63),(24),(6324).
I,(6235),(63)(25),(6532),(13),(26),(62)(35),(65)(23) ,(62),(35),
(65),(23) ,(63),(25), (6325).
I,(6234),(63)(24),(6432),(63),(24) ,(62)(34),(64)(23) ,
(62),(34),(64),(23), (63),(24), (6324).

(10) The sub groups of a group S_6 with order 24 are:

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I,(12),(13),(23),(14),(24),(34),(123),(132),(124),
(243),(134),(143),(12)(34),(13)(24),(32)(14), (1234),(1243),
(142),(234),(1324),(1342),(1423), (1432).
I,(12),(13),(23),(15),(25),(35),(123),(132),(125), (253),(135),(153),
(12)(35),(13)(25),(32)(15), (1235),(1253),(152),(235),(1325),(1352),
(1523),(1532).
I,(12),(13),(16),(23),(26),(36),(163),(136),(126),(162),(236),(263),
(16)(23),(13)(26),(36)(12), (1632),(1623),(1326)(1362),(1236),(1362)
,(1263),(1236).
I,(12),(14),(24),(15),(25),(45) ,(124),(142),(125),(254),(145),(154),(12)(45),
(14)(25), (42)(15), (1245), (1254),(152),(245),(1425),(1452) ,
(1524),(1542).
I,(12),(16),(26) ,(14),(24),(64),(126),(162),(124), (246),(164),(146),(12)(64),
(16)(24),(62)(14), (1264),(1246),(142),(264),(1624),(1642),
(1426),(1462).
I,(12),(15),(16),(25),(26),(56),(165),(156),(126), (162),(256),
(265),(16) (25),(15)(26),(56)(12), (1652),(1625),(1526),(1562),(1256)
,(1562),(1265),(1256).
I,(13),(14),(34),(15),(35),(45),(134),(143),(135), (354),(145),(154),(45),
(14)(35),(43)(15),(1345), (1354),(153),(345),(1435),(1453),(1543)
(1534), (13).
I,(14),(34),(16),(36),(46),(134),(143),(136), (364),(146),(1634),
(1643),(164),(13),(46),(14)(36), (43)(16),(1346) ,(1364),(163),
(346),(1436), (13),(1463).
I,(13),(15),(35),(16),(36),(56),(135),(153),(136),(365),(156),(165),
(13),(56),(15)(36),(53)(16), (1356),(1365),(163),(356)(1536),
(1563),(1635),(1653).
I,(14),(15), (45),(16),(46),(56),(145),(154),(146), (465),(156),(165),
(14),(56),(15)(46) ,(54)(16), (1456),(1465),(164),(456),(1546),(1564),
(1645),(1654).
I,(23),(24),(25),(34),(35),(45),(234),(243),(235),(253),(245),(254) ,
(23),(45),(24)(35),(43)(25), (2345),(2354),(253),(235)(2435),(2453),
(2534),(2543).
I,(23),(24),(26),(34),(36),(46),(234),(243),(236),(263), (246),(264),(23),
(46),(24)(36),(43)(26), (2346),(2364),(2634),(436)(463),(2463),
(2436),(2643).
I,(23),(25),(26),(35),(36),(56),(235),(253),(236),(263),(256),(265) ,(23),
(56), (25)(36),(53)(26), (2356),(2365) ,(2635),(536),(563),(2563),

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(2536),(2653).
I,(25),(24),(26),(54),(56),(46),(254),(245),(256),(265) ,(246),(264),(25),
(46),(24)(56), (45)(26), (2546),(2564),(2654),(456),(465),
(2465),(2456),(2645).
I,(35),(34),(36),(54),(56),(46),(354),(345) ,(356),(365),(346),(364),(35),(46)
,(34)(56),(45)(36), (3546),(3564),(3654),(456),(465),(3465),
(3456),(3645).

(11) The sub groups of a group S_6 with order 60 are:
(1) I,(12)(34),(125),(123), (124),(243),(134),(142),(234),(253),(135),
(153),(152),(245),(354), (143), (235),(254),(145),(154),(345),
(13)(24), (32)(14),(25)(34),(35)(24),(12)(35), (13)(25), (12)(45),
(14)(25),(42)(15), (13)(45),(14)(35),(43)(15),(15)(23),(23)(45),
(14352),(14325),(15234), (15243), (15324),(15342),(15423),
(13425),(13452),(13524),(13542), (12453),(12435),(12534),
(12543), (14235),(14253),(14532),(14523), (12345),(12354),
(13254),(13245),(132).
(2) I,(12)(34),(126),(123),(132),(124),(243),(134),(142),(234),(263),
(136) ,(163),(162),(246),(364), (143),(236), (13264) ,(13)(24),
(13624) ,(13246) (32)(14),(26)(34),(36)(24),(12)(36), (13)(26),
(12)(46),(14)(26),(42)(16), (13)(46),(14)(36),(43)(16),(32)(16),
(23)(46),(14362),(14326),(16234), (16243),(16324),(16342),
(16423),(16432),(13426),(13462),(346) ,(164) ,(13642),(12463),
(12436), (12634),(12643),(14236),(14263),(14632), (14623),
(12346),(12364),(146), (264)
(3)I,(12)(36),(125),(123),(132),(126),(263),(136),(162),(236),(253),
(135),(153),(152), (265), (356), (163),(235),(256),(165),(156),
(365),(13)(26),(32)(16),(25)(36),(35)(26),(12)(35),(13)(25),
(12)(65), (16)(25),
(62)(15), (13)(65),(16)(35),(63)(15),(32)(15),(23)(65),(16352),
(16325),(15236), (15263), (15326),(15362),(15623),(15632),
(13625),(13652),(13526), (13562),(12653),(12635),(12563),
(16235),(16253),(16532),(16523), (12365),(12356),(13256),(13265), (12536)
(4)I,(12)(64),(125),(126),(162),(124),(246),(164),(142),(264),(256) ,
(165),(156),(152),(245), (654), (143),(265),(254),(145),(154),
(645), (16)(24),(62)(14),(25)(64),(65)(24),(12)(65), (16)(25),
(12)(45), (14)(25) ,(42)(15),(16)(45),(14)(65),(46)(15),(62)(15),
(26)(45),(14652),(14625), (15264),(15246), (15624),(15642),
(15426),(15462),(16425),(16452), (16524),(16542),(12456),

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(12465),(12564), (12546),(14265),(14256), (14562),(14526), (12645),(12654),(16254),(16245)
(5)I,(16)(34),(165),(163),(136),(164),(643),(134),(142),(634),(653), (135),(153),(156),(645), (354), (143),(635),(654),(145),(154), (345) ,(13)(64),(36)(14),(65)(34),(35)(64), (16)(35), (13)(65), (16)(45), (14)(65), (46)(15),(13)(45),(14)(35),(43)(15),(36)(15), (63)(45),(14356),(14365), (15634),(15643), (15364),(15346), (15463),(15436),(13465),(13456), (13564),(13546),(16453), (16435),(16534), (16543),(14635),(14653), (14536),(14563), (16345),(16354),(13654),(13645)
(6)I,(62)(34), (625),(623), (632),(624),(243),(634),(642),(234), (253), (635),(653),(652),(245), (354), (643),(235),(254),(645), (654),(345), (63)(24),(32)(64),(25)(34),(35)(24),(62)(35), (63)(25), (62)(45), (64)(25), (42)(65),(63)(45),(64)(35),(43)(65), (32)(65),(23)(45),(64352),(64325), (65234),(65243), (65324), (65342),(65423),(65432),(63425),(63452), (63524),(63542), (62453),(62435),(62534),(62543), (64235), (64253), (64532),(64523),(62345),(62354) ,(63254),(63245).

(12) The sub groups of a group S_6 with order 120 are:
(1)I,(12),(13),(23),(14),(24),(34),(15),(35),(45),(12)(34),(125), (123),(132) ,(124),(243),(134),(142), (234),(253),(135),(153),(152), (245),(354),(143),(235),(254),(145),(154),(345),(13)(24),(32)(14), (25)(34),(35)(24),(12) (35),(13)(25),(12)(45),(14)(25),(42)(15), (13)(45),(14)(35),(43)(15),(32) (15), (23)(45),(14352),(14325), (15234),(15243),(15324),(15342),(15423),(25),(13425),(13452),(13 524), (13542),(12453),(12435),(12534),(12543), (14235),(14253), (14532),(14523),(12345), (12354), (13254),(13245),,(12) (345),(12)(354), (13)(245),(13)(254),(14)(532),(25)(134),(23)(154),(24) (135), (25)(143),(14)(235),(45)(132),(35)(142),(15)(243),(15)(234),(23) (145), (24)(153),(45)(123), (35)(124),(152)(34),(125)(34),(2354),(3245) , (3254),(4253),(2435),(2345),(1234),(1324), (1342), (1423), (1432),(1253),(1254),(1523),(1532),(14 25),(1452),(1524),(1542), (1235),(1354),(1435), (1453),(1534),(1543),(1345),(1325),(1352), (1245),(1243),(15432)
(2)I,(13),(23),(14),(24),(34),(16),(36),(46),(12)(34),(126), (123),(1243) ,(132),(124),(243),(134),(142),(234),(263),(136), (163),(162),(246),(364),(143),(236),(264),(146), (164),(346), (13)(24),(32)(14),(26)(34),(36)(24), (12),(36),(13)(26),(12)(46),

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(14)(26), (42)(16), (13)(46),(14)(36),(43)(16), (32)(16),(23)(46),
(14362),(14326), (16234),(16243),(16324),(16342), (16423),(16432),
(13426),(13462),(13624),(13642),(12463),(12436) ,(26) (12634),
(12643),(14236), (14263),(14632),(14623),(12346),(12364),(12)(13264),
(13246),(12)(346),(12)(364),(13)(246), (13)(264),
(14)(632),(26) (134),(23)(164),(24)(136),(26)(143),(14)(236),(46)(132),
(36)(142),(16) (243),(16)(234),(23)(146),(24)(163),(46)(123),(36)(124),
(162)(34),(126)(34),(2364),(3246),(3264), (4263),(2436),
(2346),(1234),(1324),(1342), ,(1423),(1432),(1263), (1264),(1623),
(1632),(1426), (1462),(1624),(1642),(1236),(1364),(1436),(14 63),
(1634),(1643),(1346),(1326),(1362),(1246) (3) I,(12),(13),
(23),(16),(26),(36),(15),(35),(45),(12)(36),(125),(123), (132),(126),
(263), (136), (162),(236),(253),(135),(153),(152),(265),(356) ,
(163),(235),(256),(165),(156), (365), (13)(26), (32)(16),(25)(36),
(35)(26), (12)(35),(13)(25),(12)(65),(16)(25), (62)(15),(13)(65),
(16)(35), (63)(15), (32),(15),(23)(65),(16352),(16325),(15236),
(15263),(15326),(15362), (15623),(15632), (13625),(13652),(13526),
(13562),(12653),(12635),(125)(36),(12563),(16235),(1 6253),(16532),
(16523),(12365),(12356),(13256), (13265), (12)(365),(12)(356),
(13)(265),(13)(256),(16)(532), (25)(136), (23)(156),(26)(135),(25)(163),
(16)(235),(65)(132),(35)(162),(1263),(25) (15)(236), (23)(165),(26)(153),
(65)(123),(35)(126),(152)(36),(125)(36),(2356),(3265),(3256),(6253),
(2635),(2365),(1236),(1326),(1362),(1623),(1632),(1253),(1256),
(1523),(1532),(1625),(1652),(1526), (1562),(1235), (1356),(1635),
(1653),(1536),(1563),(1365),(1325),(1352),(1265),(15)(26)
(4)I,(14),(24),(64),(15),(65),(45),(12)(64),(125),(126), (1245),
(1246),(162),(124), (246),(164), (142),(264),(256),(165),(156),(152),
(245),(654), (143),(265),(254),(145), (154),(645) , (16)(24),
(62)(14),(25)(64),(65)(24), (12)(65),(16)(25),(12)(45),(14)(25),(42)(15),
(16)(45),(14)(65),(46)(15), (62)(15),(26)(45),(14652),(14625),
(15264),(15246),(15624),(15642), (25) (15426),(15462),(16425),
(16452),(16524),(16542),(12456),(12465), (26)(12564),(12546), (14265),
(14256), (14562),(14526), (12645),(12654), (12) (16254),(16245),
,(12)(645),(12)(654),(16)(245),(16)(254),(14)(562), (16) (25)(164),
(26)(154),(24)(165),(25)(146),(14)(265),(45)(162),(65)(142) ,(15)(246),
(15)(264), (26)(145), (24)(156),(45)(126),(65)(124) ,(2645) (152)(64),
(125)(64),(2654),(6245), (6254),(4256), (2465), (1264),(1624), (1642),
(1426),(1462),(1256),(1254),(1526),(1562),(1425),(1452),(1524), (1542),

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(1265),(1654),(1465),(1456),(1564),(1546),(1645),(1625),(1652)
(5) I,(63),(14),(64),(34),(15),(35),(45), (16)(34), (165),(163) , (1643),(136),(164),(643), (134), (142),(634),(653),(135),(153),(156), (645),(354) ,(143),(635),(654),(145), (154),(345) ,(13)(64), (36)(14),(65)(34),(35)(64), (16)(35),(13)(65),(16)(45),(14)(65), (46)(15),(13)(45),(14)(35),(43)(15), (36)(15),(63)(45),(14356), (14365),(15634),(15643),(15364),(15346) ,(13) (15463),(15436), (13465), (13456),(13564),(13546),(16453),(16435), (16) (16534), (16543),(14635), (14653),(14536), (14563),(16345),(16354), (65) (13654),(13645),,(16)(345),(16)(354),(13)(645),(13)(654), (14)(536),(65) (134),(63)(154),(64)(135),(65)(143),(14)(635), (45)(136),(35)(146),(15) (643),(15)(634),(63)(145), (64)(153), (45)(163),(35)(164),(156)(34),(165) (34),(6354),(3645),(3654), (4653), (6435),(6345), (1634),(1364),(1346), (1463),(1436), (1653),(1654),(1563),(1536),(1465),(1456),(1564),(1546), (1635), (1354),(1435),(1453),(1534),(1543),(1345),(1365),(1356),(1645)
(6)I,(62),(63),(24),(65),(35),(45),(62)(34), (625),(623), (6243), (25) (632),(624),(243), (634),(642), (234),(253),(635),(653),(652), (245),(354), (643),(235),(254),(645),(654), (345), (63)(24), (32)(64), (25)(34),(35)(24) ,(62)(35),(63)(25),(62)(45),(64)(25), (42)(65),(63)(45),(64)(35),(43)(65), (32)(65), (23)(45), (64352),(64325),(65234),(65243),(65324),(65342) ,(34) ,(65423), (65432),(63425),(63452), (63524),(63542),(62453),(62435) ,(64) , 62534),(62543),(64235),(64253), (64532),(64523),(62345), (62354) , (23), (63254), (63245),(62)(345),(62)(354), (63)(245),(63)(254), (64)(532), (25)(634), (23)(654),(24)(635),(25)(643),(64)(235), (45)(632),(35)(642),(65) (243),(65)(234),(23)(645), (24)(653), (45)(623),(35)(624)(652)(34),(625) (34),(2354),(3245),(3254), (4253),(2435),(2345), (6234),(6324),(6342), (6423),(6432), (6253),(6254),(6523),(6532),(6425),(6452),(6524),(6542), (6235), (6354),(6435),(5453),(6534),(6543),(6345),(6325),(6352),(6245)

(13) The sub group of a group S_6 with order 360 are:
I, (12),(13),(23),(14),(15),(16),(24), (26) ,(34),(25),(56),(46), (123),(132),(124),(142),(125), (152), (126), (162),(345),(354)(134), (143),(135),(153), (136),(163),(145),(154), (346),(364), (146),(164), (156), (165), (234),(243),(235),(253),(356),(465), (236), (263),(245),(254), (246), (264),(265), (456),(256), (365), (12345),(12354),(13254),(13245), (14352),(14325), (15234),

(15243),(13542), (14523), (16243), (13642),(14623),(15263),
(15324), (15342),(15423), (15432), (13425),(13452),(13524),
(12453),(12435), (12534),(12543), (14235), (14253), (14532),
(12346),(12364), (13264),(13246),(14362),(14326),(16234),
(16324),(16342), (16423), (16432),(13426),(1362),(13624),
(12463),(12436),(12634),(12643),(14236), (14263),
(14632),(12365),(12356),(13256),(13265),(16352), (16325),
(15236),(15326), (15362), (15623),(15632),(13625),(13652),
(13526),(13562),(16523), (15246), (16542), (14526),(15643),
(13546), (12653),(12635),(12536),(12563), (16235), (16253),
(16532), (12645), (12654),(16254), (16245), (14652),(14625),
(15264),(15624),(15642), (15426), (15462),(16425),(16452),
(16524),(12456), (12465),(12564), (12546),(14265), (14256),
(14562),(16345),(16354),(13654),(13645),(14356), (14365),
(15634),(15364), (15346), (15463),(15436),(13465), (13456),
(13564),(16453),(16435), (16534), (16543),(14635), (14653),
(14536),(14563),(65243),(63542),(64523), (62345),(62354),
(63254), (63245), (64352),(64325),(65234),(65324),(65342),
(65423),(65432), (63425),(63452),(63524), (62453),(62435),
(62534),(62543),(64235), (13)(245),(13)(254), (14)(523),(14)(532),
(15)(243),(25)(134),(35)(642), (16)(243),(26)(134),(45)(623),
(15)(234), (23)(145),(23)(154), (24)(135), (24)(153),(25)(134),
(34)(125),(34)(152),(35)(124),(35)(142),(45)(123), (45)(132),
(12)(346), (12)(364), (13)(246),(13)(264),(14)(123),(14)(132),
(16)(234), (23)(146),(23)(164), (24)(136),(24)(163),(26)(134),
(34)(126), (34)(162),(36)(124),(36)(142),(46)(123), (46)(132),
(12)(365),(12)(356), (13)(265),(13)(256), (16)(123), (16)(132),
(15)(263),(25)(136),(45)(632), (15)(246),(25)(164),(34)(625),
(15)(236),(23)(165),(23)(156),(26)(135),(26)(153),(25)(163),
(36)(125),(36)(152),(35)(126),(35)(162),(65)(123), (65)(132),
(12)(645),(12)(654), (16)(245), (16)(254),(14)(126),(14)(162),
(15)(264),(26)(145), (26)(154),(24)(165), (24)(156),(25)(164),
(64)(125) ,(64)(152),(65)(124), (65)(142),(45)(126), (45)(162),
(16)(345),(16)(354),(13)(645), (13)(654),(14)(163), (14)(136),
(15)(643), (65)(134), (62)(345), (34)(652),(25)(634),(36),(46),
(15)(634),(63)(145),(63)(154),(64)(135), (64)(153), (65)(134),
(34)(165), (34)(156),(35)(164), (35)(146), (45)(163), (45)(136),
(62)(354),(63)(245),(63)(254),(64)(123),(64)(632),(65)(243),

(65)(234),(23)(645),(23)(654),(24)(635),(24)(653),(25)(634),
(123)(456),(132)(456),(124)(356),(142)(356),(125)(346),(152)(346)
,(153)(246),(164)(253),(164)(253),(142)(365), (126)(345),
(162)(345),(134)(256),(143)(256),(135)(246),(136)(245),(64253),
(64532),(12)(345), (12)(354),(163)(245),(145)(236), (154)(236),
(146)(253),(156)(243),(165)(243), (123)(465), (132)(465),
(124)(365),(125)(364),(152)(364),(126)(354),(162)(354),(134)(265),
(143)(265), (154)(263),(135)(264),(153)(264),(136)(254),
(163)(254),(145)(263),(146)(235), (164)(235), (156)(234), (165)(234)

References

[1] A. Beltran and M.J. Felipe, Some class size conditions implying solvability of finite groups, J. Group Theory, 9(6), 2006, 787-797.

[2] C. F. Gardiner, A First Course in Group Theory, Springer-Verlag, Berlin, 1997.

[3] D. R. Wilkins; Course 311: Abstract Algebra; Academic Year 2007-08; Copyright David R. Wilkins 1997-2007.

[4] D. Samaila , B. Ibrahim Abba and M. Pius Pur, On the Conjugacy Classes, Centers and Representation of the Groups Sn and Dn, Int. J. Pure Appl. Sci. Technol., 15(1), 2013, pp. 87-95.

[5] D. Samaila , M. Pius Pur and B. Ibrahim Abba, Visualizing the Homorphic Image Through Abstract Characterization of the Symmetry Group Dn, Int. J. Pure Appl. Sci. Technol., 15(2), 2013, pp. 33-42.

[6] I.N. Herstein, Abstract Algebra (3rd ed.), Prentice Hall Inc., Upper Saddle River, 1996.

[7] I.N. Herstein, Topic in Algebra, John Wiley and Sons, New York, 1975.

[8] J.B. Fraleigh, A First Course in Abstract Algebra, Addison-Wesley, London, 1992.

[9] J.J. Rotman, An Introduction to the Theory of Groups, (4th ed.), New York: Springer, 1999.

[10] R. Sulaiman, Subgroups Lattice of Symmetric Group S4; International Journal of Algebra, Vol. 6, 2012, no. 1, 29-35. [11] S. Roman, Lattice and Ordered Set, Springer, New York, 2008.

[12] W.R. Scott, Group Theory, New York: Dover,1987