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Soil erosion modeling of watershed using cubic, quadratic and quintic splines

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Abstract
Soil erosion is widespread with spatio-temporal variability and is central to the determination of sediment yield, which is vital to proper management of watersheds. We propose a relation between the Curve Number (SCS 1956) and the Sediment Yield Index (SYI) using cubic, quadratic and quintic splines in this research. Using Mohgaon watershed (part of Narmada Basin) data, the relation between observed and computed SYI is found to have a coefficient of determination (R^2) value of 0.87, 0.40 and 0.10 corresponding cubic, quadratic and quintic splines suggesting that such a relation can be used to determine SYI from the available CN value. The cubic spline was found to be the best method with respect to Absolute Prediction Error (*APE*), Integral Square Error (*ISE*), Coefficient of Efficiency (*CE*), Coefficient of Correlation (*CC*) and degree of agreement (*d*) (i.e., APE = 1.35, ISE = 3.09, CE = 62.08, CC = 79.60 and d = 0.99). The quintic spline (with an average value of APE = 19.59, ISE = 7.84, CE = − 165.73, CC = 19.30 and d = 0.26) and the quadratic spline (with an average value of APE = 20.99, ISE = 8.92, CE = − 199.90, CC = 8.95 and d = 0.15) ranked as the 2nd and the 3rd best methods, respectively.

Keywords Sediment yield index · Cubic/quadratic/quintic spline · Mohgaon watershed · Soil erosion

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1 Introduction

Soil erosion is a significant issue in virtually all countries of the world. Almost 1964.4 M ha of land is affected by human-instigated debasement (Meshram et al. 2017; Meshram and Meshram 2020). Of this amount, about 1903 M ha land is degraded by soil erosion by water and remainder by the wind erosion. In India, of 329 M ha land, around 167 M ha is influenced by water and wind erosion. The land influenced by water erosion is evaluated to be around 113.3 M ha (Ministry of Agriculture 1972; Meshram et al. 2019). These figures indicate that land management needs urgent attention.

Proper watershed management programs require quantitative values of soil loss. The soil loss varies from one watershed to another. It is desirable to identify critical watersheds, prioritize watersheds and then undertake needed measures for soil and water protection (Meshram and Meshram 2020).

Sediment yield from a catchment is one of the principal bases for the prioritization of watersheds prone to soil disintegration (Brahim et al. 2020). This requires continuously observing sediment sample at the watershed outlet. However, continuous measurements of soil loss are not available for most watersheds, especially in developing countries, such as India (Meshram et al. 2020). Since soil loss measurements are expensive, watersheds can be prioritized and measurements can be undertaken for developing prediction models which can be applied to un-gauged watersheds.

The watershed selected for investigation gets > 80% of the precipitation in the rainy season (June–September) (Meshram et al. 2018). Due to undulated topography, a significant part of the water streams out rapidly and results in soil erosion and poorly recharges groundwater aquifers. The light finished and penetrable soils are easily erodible and hold restricted amounts of water in the root zone. For the most part precipitation ends during the last week of September or the first week of October, and accordingly crop flowering and development suffer significantly because of low moisture and hence yield is affected (Singh et al. 2020).

Sediment yield and runoff data are required for watershed protection of soil and water assets. The Curve Number (CN) speaks to the overflow potential and shows spatial and worldly variety. The Soil Conservation Service-Curve Number (SCS-CN) technique (1954) process the surface runoff for the certain precipitation events from small watersheds (SCS 1956, 1985), which is required for calculation of soil disintegration/erosion. The watershed susceptibility of soil disintegration and silt yield is displayed by utilizing the idea of SYI of All India Soil &Land Use utilizing the information of Mohgaon Watershed (India). It is known that silt to a large degree depends upon precipitation-runoff and watershed conditions characterized by runoff CN. The greater the runoff CN, the greater will be the sediment yield in agrarian watersheds and the other way around. Since CN speaks to the runoff delivering capability of a watershed and sediment yield index, the capability of residue yield, it is very legitimate that these two parameters, i.e., sediment yield index and curve number, should show some connection among them (Gajbhiye et al. 2014 2015; Meshram and Powar 2017a, b; Meshram et al. 2017).

Spline functions which are piecewise polynomial functions are used nowadays. They are suited for the estimation of experimental data or design curve experiments (Rice 1969). Piecewise polynomials of grade n are defined by the first $n-1$ derivatives which appear in the focus of the joining. The quantity and degrees of the equation parts and the number and location of the nodes may alter in distinct situations. A big amount of research have been performed at regional and national level on the use of cubic, quadratic and quintic spline (Herriot and Reinsch 1976; Yang and Wang 1994; Holnicki 1996; Kumar and Srivastava 2009; Christara et al. 2010; Tianxiang and Hongxia 2012; Han 2015; Wu and Zhang 2014; Alayed et al. 2016; Tariq and Akram 2016; Luo et al. 2016; Wong 2017; Lang 2017; Moghaddam et al. 2017; Li and Wong 2019; Gülüm et al. 2019; Khalid et al. 2019; Psiaki et al. 2019; Černá and Finěk 2020; Moghaddam et al. 2021). Some researchers have applied contemporary modeling tools in water resources engineering (Wu and Chau 2006; Chau 2007; Wang et al. 2014; Taormina et al. 2015; Gholami et al. 2015; Chen et al. 2015).

Spline functions have been implemented to measure approximately the SYI value for a given CN that can contribute significantly to the watershed management sector. Since the spline approximation theory states that the best approximants are cubic splines. Therefore, we applied these splines and computed splines of various orders and found that, relative to other quadratic and quintic splines, the cubic splines gave approximations very similar to the actual values. Although the cubic spline is most frequently used in spline feature estimates, it is also considered suitable for quadratic spline. The spline function is increasingly used in medicine, agriculture, engineering and various sciences, but has experienced limited use in soil erosion. Therefore, the purpose of this research was to apply three methods of splines; cubic, quadratic and quintic, for the index of computational sediment yield.

2 Material and methods

2.1 Study area

The Burhner River is the main tributary of Mohgaon Watershed which ascends in the Maikala range in the Mandla district of Madhya Pradesh (India) with elevation of around 900 m. Mohgaon Watershed with an area of 3978 km^2 lies between latitude of 22°32′N and longitude of 81°22′E (Fig. 1). The weather of the basin is delegated sub-tropical and sub humid with a normal yearly precipitation of 1,547 mm. The watershed region contains both smooth and rippling lands secured with wood and urban lands. Soils are generally red and silty clay loam. Cultivated and forest land share almost 53 and 12% of the catchment region, correspondingly (Gajbhiye et al. 2014). For model development, we used the data of SYI and CN from the previous study of Meshram and Powar (2017a, b). The soil erosion problem in the Mohgaon watershed shown in Fig. 2.

2.2 Importance and reference of spline functions in approximation

At the early stage, the approximation process was quite rough, as when two values were available (e.g., the SYI) which corresponded to two separate domain points (representing the CN), the approximate value in between two domain CNs used to be a continuous function obtained through the calculation of an average two values.

The polynomials were also found to be the best approximate feature because they can be measured readily and distinguished. The easy principle of matrix interpolation on which numerical assessment is practically based can be discovered in numerical assessment literature (Davis 1953). Later Weierstrass in 1885 established a very strong result supporting polynomials as good approximates for continuous functions. But the question is whether the approximate polynomial $P_n(x)$ converges always or not?

The answer is that $P_n(x)$ may not be convergent in general. In early nineteen century Me'ray and later Runge looked at the meromorphic function $f(x) = 1/1 + x^2$ and

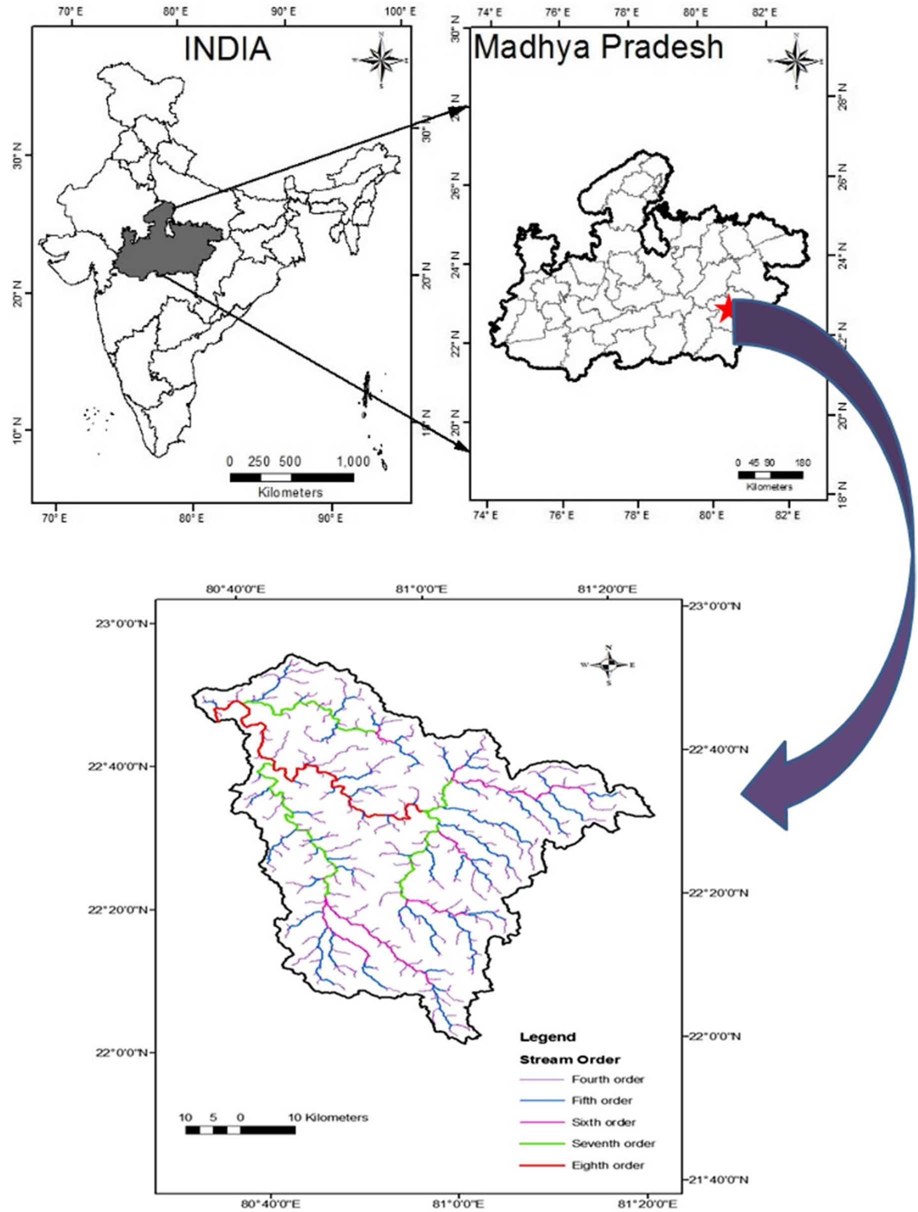

Fig. 1 Location map of the study area

investigated that if $P_n(x)$ interpolate to f at $n+1$ equidistant point of the interval $|x| \leq 5$, P_n converges to f only for the interval $|x| \leq 3.63$ and diverges outside this interval. The above assertion and many such observations of non-convergence lead to think mathematician for some alternate approximates. The progress of software science makes it easy to store features on the desktop and they have good mathematical characteristics like:

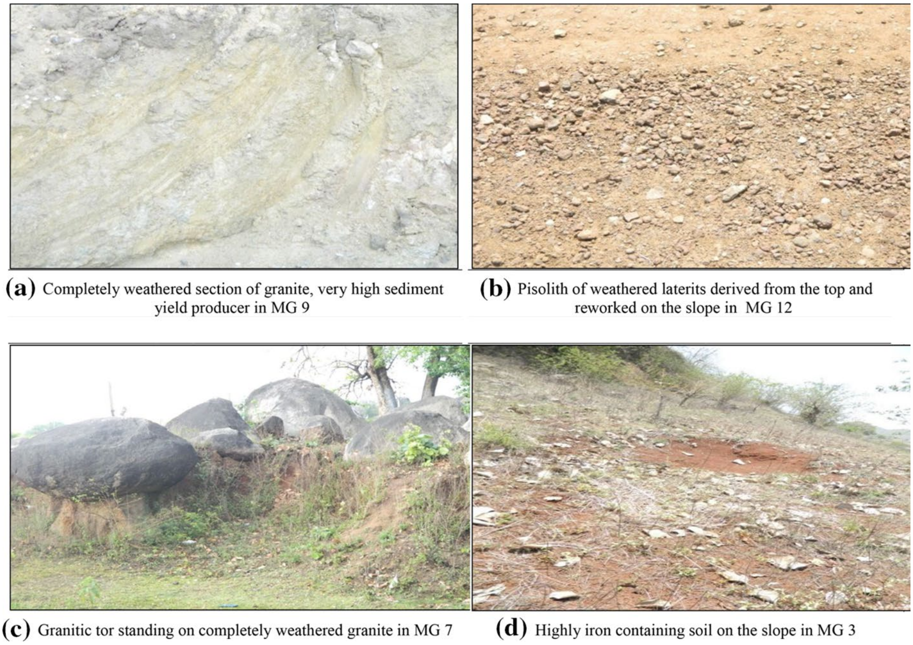

Fig. 2 Soil erosion problem in the Mohgaon (MG) watershed

1. Strong approximation power
2. Computational convenience
3. Sufficient degree of smoothness

Play a significant of one such major approach could be traced back to the pioneering work of Schoenberg (1968). The fundamental concept is to apply piecewise polynomial (PP) functions that are called Splines with a certain degree of smoothness at the joints. One obvious benefit of using PP functions rather than variables is that a greater degree of flexible is accomplished by splines without charging for further computing complexities that are related to higher-grade variables. Spline is a French word; their significance is an architectural device of tiny bends used for the layout of railway tracks by engineers.

2.3 Primilaries of splines

In order to develop a relationship between SYI and CN, the proposed approach is outlined as being taken after: (1) Estimation of sub-watershed wise SYI and CN, (2) CN of all sub watersheds is organized in increasing order of CN, which will give the interval of CN for spline development, (3) As in our investigation, 15 sub-watersheds are there, so eight (2, 4, 8, 9, 10, 11, 12, 15) were chosen for the development of splines and the remaining were utilized for the approval of splines, (4) From the CN value and spline coefficients, compute the Sediment Yield Index which corresponds to the interval of CN. These are the computed SYI values, (5) The computed SYI (SYIc) was compared with the observed SYI (SYIo) which was derived from the AISLUS method. In addition, compiling the soil loss map according to the following methodological flowchart (Fig. 3).

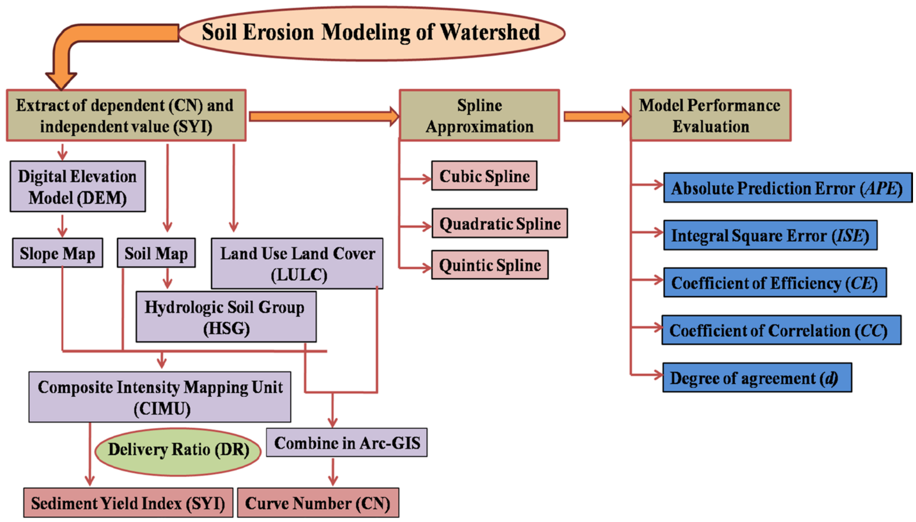

Fig. 3 Methodology adopted for the realization of the soil erosion modeling in the Mohgaon watershed

The computation of SYI corresponding to any region is based on many factors like; slope, watershed area, soil type and land use land cover information. These data have been extracted from the satellite information, soil maps and SRTM map with the help of GIS environment. It is well known that the computation of SYI is quite tedious. Hence considering the curve number (CN) in the mesh of domain the corresponding quadratic, quintic and cubic spline approximation has been obtained for SYI. In the literature of approximation theory (De Boor 1978), it has been mentioned that the cubic spline are the best approximants. We have computed quadratic, quintic and cubic spline for the Mohgaon Watershed data and concluded practically that cubic splines are the best approximants. The main objective of this comparison to make sure that in case of watershed management also, the cubic approximants behave nicely and are very close to the actual values.

2.3.1 Cubic spline (Gülüm et al. 2019)

The $[\alpha, \beta]$ interval on the actual route and outline a divider of it as follows:

$$\alpha = z_1 < z_2 < \cdots < z_n = \beta$$

The z_i's are the nodal points and for $i = 1, 2, \ldots, n$ at every z_i, $\delta(z_i)$ is given. Our goal is to build piecewise cubic interpolant f to δ. On every sub-interval, we demonstrate $p_i(x)$ (a piecewise polynomial) such that

$$f(z) = p_i(z) \text{ for certain } p_i(z) \in \mathbb{P}_4 (i = 1, 2, \ldots, n-1).$$

The following circumstances have been executed on the ith polynomial piece p_i:

$$\begin{aligned} p_i(x_i) &= \delta(z_i),\ p_i(z_{i+1}) = \delta(z_{i+1}) \\ p_i'(z_i) &= s_i,\ p_i'(z_{i+1}) = s_{i+1} \end{aligned} \quad (1)$$

$$i = 1, 2, \ldots, n-1$$

Here $s_1, \ldots, s_n$ are free factors. f (approximate function) approves with δ at $z_1, \ldots, z_n$, constant up-to the first order on $[\alpha, \beta]$.

For computing the constants of the ith p_i (polynomial piece), we use its Newton form:

$$p_i(z) = d_i(z_i) + (z - z_i)[z_i, z_i]d_i + (z - z_i)^2[z_i, z_i, z_{i+1}]d_i + (z - z_i)^3(z - z_{i+1})[z_i, z_i, z_{i+1}, z_{i+1}]d_i \tag{2}$$

Its coefficients have been determined from the distributed difference for d_i using the input data:

Knots	Data	1st divided difference $[,]d_i$	2nd divided difference $[,,]d_i$	3rd divided difference $[,,,]d_i$
z_i	$d(z_i)$	s_i	$([z_i, z_{i+1}]\delta - s_i)/\Delta z_i$	$(s_{i+1} + s_i - 2[z_i, z_{i+1}]\delta)/(\Delta z_i)^2$
z_i	$d(z_i)$	$[z_i, z_{i+1}]\delta$	$(s_{i+1} - [z_i, z_{i+1}]\delta)/\Delta z_i$	
z_{i+1}	$d(z_{i+1})$	s_{i+1}		
z_{i+1}	$d(z_{i+1})$			

These indications that, in terms of lifted powers $(z - z_i)^r$,

$$d_i(z) = c_{1,i} + c_{2,i}(z - z_i) + c_{3,i}(z - z_i)^2 + c_{4,i}(z - z_i)^3 \tag{3}$$

with

$$c_{1,i} = d_i(z_i) = \delta(z_i),$$

$$c_{2,i} = d_i'(z_i) = s_i,$$

$$c_{3,i} = \frac{d_i''(z_i)}{2} = [z_i, z_i, z_{i+1}]d_i - \Delta z_i\left([z_i, z_i, z_{i+1}, z_{i+1}]_i'\right) = ([z_i, z_{i+1}]\delta - s_i)/\Delta z_i - c_{4,i}\Delta z_i$$

$$c_{4,i} = d_i'''(z_i)/6 = (s_{i+1} + s_i - 2[z_i, z_{i+1}]\delta)/(\Delta z_i)^2$$

$$d_{i-1}''(z_i) = d_i''(z_i), \qquad i = 2, 3, \ldots, n-1 \tag{4}$$

or

$$2c_{3,i-1} + 6c_{4,i-1}\Delta z_{i-1} = 2c_{3,i} \tag{5}$$

or

$$2\{([z_{i-1}, z_i]\delta - s_{i-1})/\Delta z_{i-1} - c_{4,i-1}\Delta z_{i-1}\} + 6c_{4,i-1}\Delta z_{i-1} = 2\{([z_i, z_{i+1}]\delta - s_i)/\Delta z_i - c_{4,i}\Delta z_i\}$$
$$\text{Or } 2([z_{i-1}, z_i]\delta - s_{i-1})/\Delta z_{i-1} + 4c_{4,i-1}\Delta z_{i-1} = 2([z_i, z_{i+1}]\delta - s_i)/\Delta z_i - 2c_{4,i}\Delta z_i \tag{6}$$

or

$$2\left(\left[z_{i-1}, z_i\right]\delta - s_{i-1}\right)/\Delta z_{i-1} + 4(s_i + s_{i-1} - 2\left[z_{i-1}, z_i\right]\delta)/\Delta z_{i-1} \\ = 2\left(\left[z_i, z_{i+1}\right]\delta - s_i\right)/\Delta z_i - 2\left(s_{i+1} + s_i - 2\left[z_i, z_{i+1}\right]\delta\right)/\Delta z_i \tag{7}$$

or

$$s_{i-1}\Delta z_i + s_i 2\left(\Delta z_{i-1} + \Delta z_i\right) + s_{i+1}\Delta z_{i-1} = \beta_i \tag{8}$$

with

$$\beta_i = 3(\Delta z_i\left[z_{i-1}, z_i\right]\delta + \Delta z_{i-1}\left[z_i, z_{i+1}\right]\delta, \quad i = 2, \dots, n-1 \tag{9}$$

The rest parameters s_1 and s_n have been chosen such that $s_1 = 0$ and $s_n = 0$.

2.3.2 Quadratic spline (Moghaddam et al. 2017)

To determine a numerical model of a quadratic spline, assume that the data are:
$\left\{\left(x_i f_i\right)\right\}_{i=0}^{n}$ wherever, as for linear splines,

$$a = x_0 < x_1 < \cdots x_n = b, \quad h \equiv \max\left|x_i - x_{i-1}\right|,$$

A quadratic spline $\mathcal{S}_{2,n}(x)$ is a C^1(C^1 continuity) piecewise quadratic polynomial. This means that:

$\mathcal{S}_{2,n}(x)$ is piecewise quadratic, that is, among successive knots x_i

$$\mathcal{S}_{2,n}(x) = \begin{cases} p_1(x) = a_1 + b_1 x + c_1 x^2, \ x \in \left[x_0, x_1\right] \\ p_2(x) = a_2 + b_2 x + c_2 x^2, \ x \in \left[x_1, x_2\right] \\ \vdots \\ p_n(x) = a_n + b_n x + c_n x^2, \ x \in \left[x_{n-1}, x_n\right] \end{cases} \tag{10}$$

$\mathcal{S}_{2,n}(x)$ is C^1; that is,$\mathcal{S}_{2,n}(x)$ is reliable and has steady first subordinate anywhere in the meantime [a, b], exactly, at the bunches.

For $\mathcal{S}_{2,n}(x)$ we would also like to have to be an interpolatory quadratic spline.

$\mathcal{S}_{2,n}(x)$ interpolate the statistics, that is,

$$\mathcal{S}_{2,n}\left(x_i\right) = f_i, \quad i = 0, 1, \dots \dots .., n \tag{11}$$

Inside each $\left(x_{i-1}, x_i\right)$ interval, the comparing quadratic polynomial is nonstop and has constant subsidiaries of all requests. Subsequently, $\mathcal{S}_{2,n}(x)$ or one of its subsidiaries can be spasmodic just at a bunch. Give it a chance to be watched that capacity $\mathcal{S}_{2,n}(x)$ has two quadratic pieces inside knot x_i; to one side of x_i it is a quadratic $\mathcal{P}_i(x)$ though correcting it as a quadratic $\mathcal{P}_{i+1}(x)$.

In this manner, an important and adequate condition for $\mathcal{S}_{2,n}(x)$ for these two quadratic polynomials, having a constant first subsidiary is an internal event to co-ordinate in the primary lower estimate. So we have an agreement of conditions of flatness: in every knot inside,

$$\mathcal{P}'_{i+1}\left(x_i\right) = \mathcal{P}'_i\left(x_i\right), \qquad i = 1, 2, \dots \dots, n. \tag{12}$$

We also have an agreement for conditions of interpolation to attach data: i.e.,

$$f_{i-1} = \mathcal{P}_i(x_{i-1}), f_i = \mathcal{P}_i(x_i), i = 1, 2, \ldots \ldots, n. \tag{13}$$

In this line, forcing $\mathcal{S}_{2,n}(x)$ to be continuous in the nodes also compounds the inclusion scenario. Given that all n quadratic parts have three unknown variables, 3n unexploded numbers are involved in our analysis of the ability $\mathcal{S}_{2,n}(x)$. Granting concordance between main auxiliary powers $(n-1)$ and immediate coefficient constraints and entry powers an additional 2n imperative. Thus, in the 3n dark equations, there are a total of $3n-1$ immediate imperatives. In all, we require 1 gradually (directly) limiting that there is an indiscernible amount of circumstances from requests.

2.3.3 Quintic spline (Tariq and Akram 2016)

Let $x_i = i\hbar\left(n > 0, \hbar = L_n, \quad i = 0, 1, \ldots .n\right)$ be network ideas of the constant divider of [0, b] into the $\left[x_{i-1}, x_i\right]$ subintervals. Let $u(x)$ be an sufficiently differentiable ability characterized on [0,b] and $\mathcal{S}(x)$ be a quintic spline approximation to $u(x)$. Reflect that every quintic polynomial spline piece $\mathcal{P}_i(x)$ has the accompanying structure:

$$\mathcal{P}_i(x) = a_i\left(x - x_i\right)^5 + b_i\left(x - x_i\right)^4 + c_i\left(x - x_i\right)^3 + d_i\left(x - x_i\right)^2 + e_i\left(x - x_i\right) + f_i \tag{14}$$

$i = 0, 1, \ldots \ldots, n-1$, Beside with the prerequisite that

$$\mathcal{P}_i(x) \in c^4[0, b] \tag{15}$$

$$\mathcal{S}(x) = \mathcal{P}_i(x), \forall x\left[x_i, x_{i+1}\right], \quad i = 0, 1, \ldots \ldots, n-1 \tag{16}$$

To improve the stability relations among the estimates of spline,

$$\begin{aligned} P_i(x_i) &= \mathcal{S}_i, P_i(x_{i+1}) = \mathcal{S}_{i+1} \\ P_i^2(x_i) &= M_i, P_i^2(x_{i+1}) = M_{i+1} \\ P_i^4(x_i) &= F_i, P_i^4(x_{i+1}) = F_{i+1} \end{aligned} \tag{17}$$

It is clear that in relation to $\mathcal{S}_i s$ and any three subordinates, the spline can be formed at the boundaries of each subinterval. The numbers described in Eq. (5) are to characterize the spline to $\mathcal{S}_i s$ and $F_i s$ as calculated:

$$\begin{aligned} a_i &= \frac{1}{120h}\left(F_{i+1} - F_i\right) \\ b_i &= \frac{1}{24}F_i \\ C_i &= 16h\left(M_{i+1} - M_i\right) - h36\left(F_{i+1} + 2F_i\right) \\ d_i &= 12M_i \\ e_i &= 1h\left(\mathcal{S}_{i+1} - \mathcal{S}_i\right) - h6\left(M_{i+1} + 2M_i\right) + h360\left(7F_{i+1} + 8F_i\right) \\ f_i &= \mathcal{S}_i \end{aligned} \tag{18}$$

Applying the 1st and 3rd subsidiary progressions at the knots,

$$P_i^\rho(x_i) = P_{i-1}^\rho(x_i) \tag{19}$$

where $\rho = 1$ and 3, the accompanying useful relations are gotten:

$$M_{i-1} + 4M_i + M_{i+1} = 12h\left(\mathcal{S}_{i-1} - 2\mathcal{S}_i + \mathcal{S}_{i+1}\right) + h260\left(7F_{i+1} + 16F_i + 7F_{i-1}\right). \quad (20)$$

$$M_{i-1} - 2M_i + M_{i+1} = h26\left(F_{i+1} + 4F_i + F_{i-1}\right) \quad (21)$$

Using conditions Eqs. (11) and (12), the accompanying consistency connection in regards to the fourth subordinate of spline F_i and $S_i i = 0, 1, \dots \dots .., n$, is derived:

$$\mathcal{S}_{i+2} - 4\mathcal{S}_{i+1} + 6\mathcal{S}_i - 4\mathcal{S}_{i-1} + \mathcal{S}_{i-2} = \frac{h^4}{120}\left(F_{i+2} + 26F_{i+1} + 66F_i + 26F_{i-2} + F_{i-2}\right) \quad (22)$$

The two end conditions are

$$-2\mathcal{S}_0 + 5\mathcal{S}_1 - 4\mathcal{S}_2 + \mathcal{S}_3 = -h2M_0 + \frac{h^4}{120}\left(18F_0 + 65F_1 + 26F_2 + F_3\right) \quad (23)$$

And $\mathcal{S}_{n-3} - 4\mathcal{S}_{n-2} + 5\mathcal{S}_{n-1} - 2\mathcal{S}_n = -h^2M_n + \frac{h^4}{120}(F_{n-3} + 26F_{n-2} + 65F_{n-1} + 18F_n)$ (24)

2.4 Performance evaluation of spline

This study utilized five assessment criteria to survey the performance of cubic, quadratic and quintic splines, including the Absolute Prediction Error (*APE*), Integral Square Error (*ISE*), Coefficient of Efficiency (*CE*), Coefficient of Correlation (*CC*) and degree of agreement (*d*) which can be communicated as follows:

$$\text{APE} = \frac{\sum_{i=1}^{n}(Mi - Ci)}{\sum_{i=1}^{n} Mi} * 100 \quad (25)$$

$$\text{ISE} = \frac{\left[\sum_{i=1}^{n}(Mi - Ci)^2\right]^{0.5}}{\sum_{i=1}^{n} Mi} * 100 \quad (26)$$

$$\text{CE} = 1\frac{\sum_{i=1}^{n}(Mi - Ci)^2}{\sum_{i=1}^{n}(Mi - m)^2} * 100 \quad (27)$$

$$\text{CC} = \frac{\sum_{i=1}^{n}(Mi - m)(Ci - c)}{\sqrt{\sum_{i=1}^{n}(Mi - m)^2\left(\sum_{i=1}^{n} Ci - c\right)^2}} * 100 \quad (28)$$

$$d = 1 - \frac{\sum_{i=1}^{n}(Mi - Ci)^2}{\sum_{i=1}^{n}[\left(\left|C_i - c\right|\right) + \left(\left|M_i - m\right|\right)]^2} * 100 \quad (29)$$

In which the value of C_i, M_i, c and m, at the corresponding time, calculated/modeled, and mean value of the observed SYIs, is calculated and observed, independently of that value. The designs with ISE < 15%, APE < 35%, CE > 60%, d closer to 1 and CC > 0.75 are deemed to be of adequate exactness.

3 Splines construction

AISLUS proposed an empirical relationship between SYI and area & delivery ratio. Different approaches were proposed for empirical relationship between delivery ratio and morphological characteristics of catchment such as catchment area, average relief or slope. These models are popularly employed because of their simplicity and easily available data.

The SYI was simplified utilizing the Mohgaon sub-watersheds data (Table 1), in perspective of the spline. The sub-watersheds 2, 4, 8, 9, 10, 11, 12, 15 were utilized for spline development and rest of the watersheds utilized for approval/validation of splines estimates.

We constructed the splines as explained in Sect. 3 for the Mohgaon watershed as follows:

As a nodal point consider CN value and the data point as SYI. Now the nodal points (CN) were set as monitors:

$$x_1 < x_2 < x_3 < x_4 < x_5 < x_6 < x_7 < x_8$$

Using specified data at the knots $\delta(x_1), \delta(x_2), \delta(x_3), \delta(x_4), \delta(x_5), \delta(x_6), \delta(x_7), \delta(x_8)$ we constructed a piecewise quadratic, quintic and cubic polynomial.

3.1 Cubic spline

We find out the polynomial pieces as demonstrated in Sect. 3. The piecewise cubic polynomials are, as shown in Fig. 4:

Table 1 Mohgaon Watershed data set utilized for cubic/quadratic/quintic spline

Sub-watershed	SYI	CN
MG 1	1252.84	61.65
MG 2	973.76	68.04
MG 3	1109.84	56.65
MG 4	965.63	65.64
MG 5	943.09	73.51
MG 6	1054.65	55.28
MG 7	1322.99	62.48
MG 8	938.59	74.96
MG 9	1259.42	56.17
MG 10	1148.61	78.28
MG 11	846.58	62.68
MG 12	1337.36	59.38
MG 13	956.43	54.49
MG 14	915.91	61.47
MG 15	1052.09	54.35

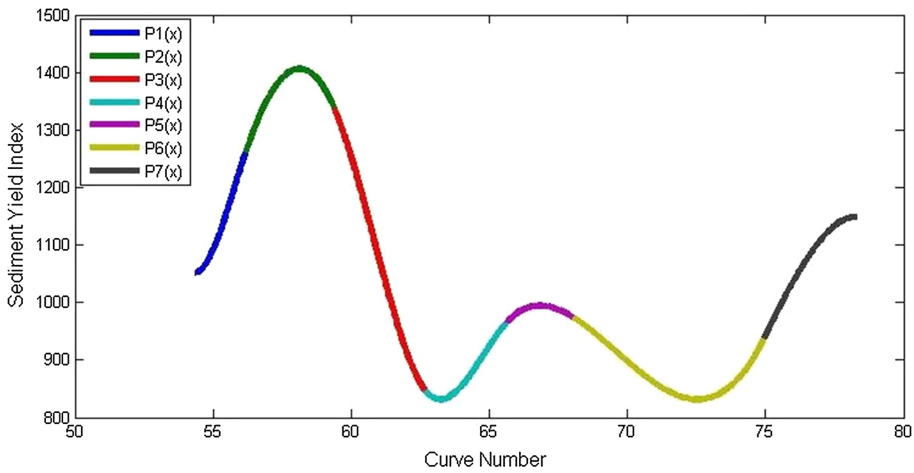

Fig. 4 Approximation of given dataset by cubic splines

For the interval [54.35–56.17]

$$p_1(x) = 1052.09 + 0(x - 54.35) + 109.41(x - 54.35)^2 - 25.73(x - 54.35)^3 \tag{30}$$

For the interval [56.17–59.38]

$$p_2(x) = 1259.42 + 142.62(x - 56.17) - 31.05(x - 56.17)^2 - 1.81(x - 56.17)^3 \tag{31}$$

For the interval [59.38–62.68]

$$p_3(x) = 1337.36 - 112.73(x - 59.38) - 48.50(x - 59.38)^2 + 11.39(x - 59.38)^3 \tag{32}$$

For the interval [62.68–65.64]

$$p_4(x) = 846.58 - 60.65(x - 62.68) + 64.28(x - 62.68)^2 - 10.20(x - 62.68)^3 \tag{33}$$

For the interval [65.64–68.04]

$$p_5(x) = 965.63 + 51.70(x - 65.64) - 26.32(x - 65.64)^2 + 2.58(x - 65.64)^3 \tag{34}$$

For the interval [68.04–74.96]

$$p_6(x) = 973.76 - 30.06(x - 68.04) - 7.74(x - 68.04)^2 + 1.64(x - 68.04)^3 \tag{35}$$

For the interval [74.96–78.28]

$$p_7(x) = 938.59 + 98.44(x - 74.96) - 2.14(x - 74.96)^2 - 2.55(x - 74.96)^3 \tag{36}$$

3.2 Quadratic spline

We find out the polynomial pieces as demonstrated in Sect. 3. The piecewise quadratic polynomials are, as shown in Fig. 5:

For the interval [54.35–56.17]

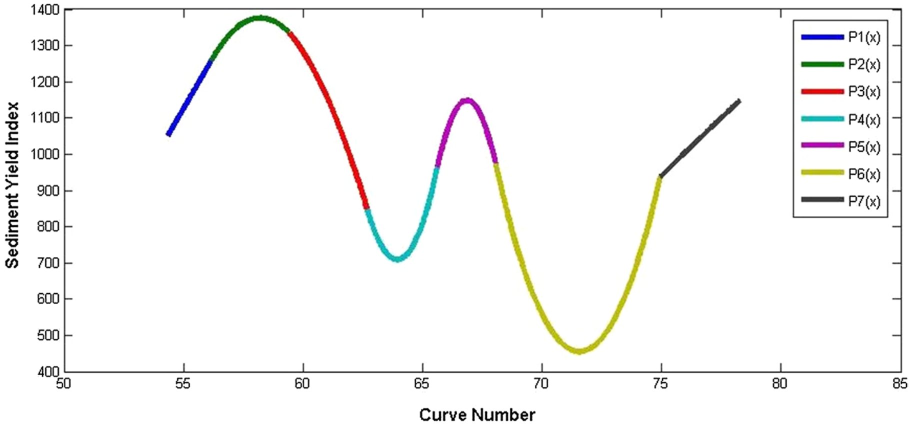

Fig. 5 Approximation of given dataset by quadratic splines

$$p_1(x) = -5139.33 + 113.92x + 0x^2 \tag{37}$$

For the interval [56.17–59.38]

$$p_2(x) = -93242.62 + 3250.94x - 27.92x^2 \tag{38}$$

For the interval [59.38–62.68]

$$p_3(x) = -71483.44 + 2529.03x - 21.94x^2 \tag{39}$$

For the interval [62.68–65.64]

$$p_4(x) = 361575.43 - 11289.06x + 88.29x^2 \tag{40}$$

For the interval [65.64–68.04]

$$p_5(x) = -554116.29 + 16611.36x - 124.24x^2 \tag{41}$$

For the interval [68.04–74.96]

$$p_6(x) = 214837.27 - 5991.62x + 41.86x^2 \tag{42}$$

For the interval [74.96–78.28]

$$p_7(x) = -8087.82 + 175.15x - 0.73x^2 \tag{43}$$

3.3 Quintic splines

We evaluate the polynomial pieces as demonstrated in Sect. 3. The piecewise quintic polynomials are, as illustrated in Fig. 6:

For the interval [54.35–59.38]

$$p_1(x) = -0.9297 - 20.1096x - 269.2490x^2 + 13.2029x^3 - 0.2143x^4 + 0.0012x^5 \tag{44}$$

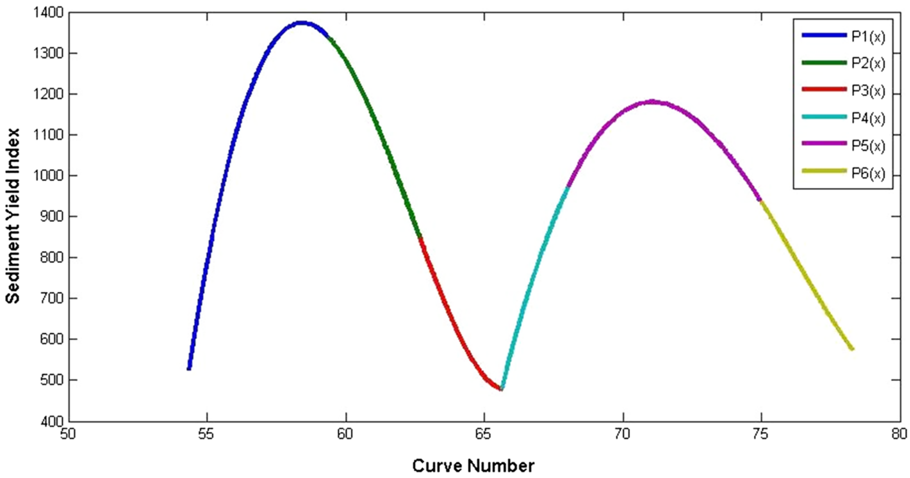

Fig. 6 Approximation of given dataset by quintic splines

For the interval [59.38–62.68]

$$p_2(x) = -0.7156 - 17.6459x - 269.4153x^2 + 13.2071x^3 - 0.2144x^4 + 0.0012x^5 \quad (45)$$

For the interval [62.68–65.64]

$$p_3(x) = -0.5895 - 15.9966x - 269.5206x^2 + 13.2096x^3 - 0.2144x^4 + 0.0012x^5 \quad (46)$$

For the interval [65.64–68.04]

$$p_4(x) = 0.0234 - 3.2023x - 119.7290x^2 + 4.8090x^3 - 0.0640x^4 + 0.0003x^5 \quad (47)$$

For the interval [68.04–74.96]

$$p_5(x) = -0.2674 - 6.8065x - 119.5169x^2 + 4.8043x^3 - 0.0639x^4 + 0.0003x^5 \quad (48)$$

For the interval [74.96–78.28]

$$p_6(x) = -0.4616 - 10.1058x - 119.3407x^2 + 4.8008x^3 - 0.0639x^4 + 0.0003x^5 \quad (49)$$

4 Splines validation and inter-comparison of splines

For validation of the spline methods applied, the parameters provided in Tables 2, 3, 4 and CN values were used in equations (Eqs. 30–49) for computing SYI. This figured sediment yield index named as computed SYI (SYI_C) was matched with the conventionally obtained SYI using AISLUS method named as observed SYI (SYIo). The values of calculated SYI were shown in Figs. 7, 8 and 9 against the SYI observed. The model assessment measures are appeared in Table 5. Both value SYI observed and computed were analyzed through a perfect fit line. The model factor values for Mohgaon watershed are given in Tables 2, 3, 4.

To compare the applied splines (quadratic/quintic/cubic), Table 5 indicates the statistical criteria of the applied splines. It was detected that the cubic spline anticipated better in approximating SYI from runoff CN compared to quadratic and quintic splines. The assessment measures, viz., d, APE, CC, ISE and CE were within the permissible limits in the cubic spline as recommended by Pyasi and Singh (2004).

As expected, the cubic spline produced the best accuracy with respect to all performance criteria (i.e., APE = 1.35, ISE = 3.09, CE = 62.08, CC = 79.60 and d = 0.99). This clearly shows that the cubic spline could be a suitable tool for sediment prediction at daily scale. Appropriately the quintic spline (with an average value of APE = 19.59, ISE = 7.84, CE = − 165.73, CC = 19.30 and d = 0.26) was ranked second and the quadratic spline (with

Table 2 Values of the coefficient and constructed cubic spline

CN	Coefficient				Splines
	C_1	C_2	C_3	C_4	
54.35–56.17	1052.09	0.00	109.41	− 25.73	$p_1(x)$
56.17–59.38	1259.42	142.62	− 31.05	− 1.81	$p_2(x)$
59.38–62.68	1337.36	− 112.73	− 48.50	11.39	$p_3(x)$
62.68–65.64	846.58	− 60.65	64.28	− 10.20	$p_4(x)$
65.64–68.04	965.63	51.70	− 26.32	2.58	$p_5(x)$
68.04–74.96	973.76	− 30.06	− 7.74	1.64	$p_6(x)$
74.96–78.28	938.59	98.44	− 2.14	− 2.55	$p_7(x)$

Table 3 Values of the coefficient and constructed quadratic spline

CN	Coefficient			Splines
	C_1	C_2	C_3	
54.35–56.17	0.0000	113.9176	− 5139.3306	$p_1(x)$
56.17–59.38	− 27.9244	3250.9406	− 93,242.6232	$p_2(x)$
59.38–62.68	− 21.9380	2529.0303	− 71,483.4369	$p_3(x)$
62.68–65.64	88.2893	− 11,289.0573	361,575.4289	$p_4(x)$
65.64–68.04	− 124.2368	16,611.3608	− 554,116.2918	$p_5(x)$
68.04–74.96	41.8639	− 5991.6247	214,837.2742	$p_6(x)$
74.96–78.28	− 0.7302	175.1496	− 8087.8233	$p_7(x)$

Table 4 Values of the coefficient and constructed quintic spline

CN	Coefficient						Splines
	C_1	C_2	C_3	C_4	C_6	C_7	
54.35–59.38	0.0012	− 0.2143	13.2029	− 269.2490	− 20.1096	− 0.9297	$p_1(x)$
59.38–62.68	0.0012	− 0.2144	13.2071	− 269.4153	− 17.6459	− 0.7156	$p_2(x)$
62.68–65.64	0.0012	− 0.2144	13.2096	− 269.5206	− 15.9966	− 0.5895	$p_3(x)$
65.64–68.04	0.0003	− 0.0640	4.8090	− 119.7290	− 3.2023	0.0234	$p_4(x)$
68.04–74.96	0.0003	− 0.0639	4.8043	− 119.5169	− 6.8065	− 0.2674	$p_5(x)$
74.96–78.28	0.0003	− 0.0639	4.8008	− 119.3407	− 10.1058	− 0.4616	$p_6(x)$

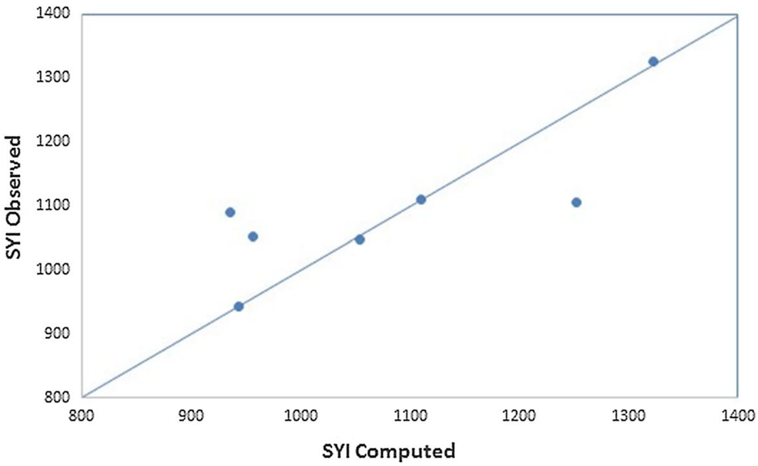

Fig. 7 Scatter plot between actual and predicted SYI by cubic splines

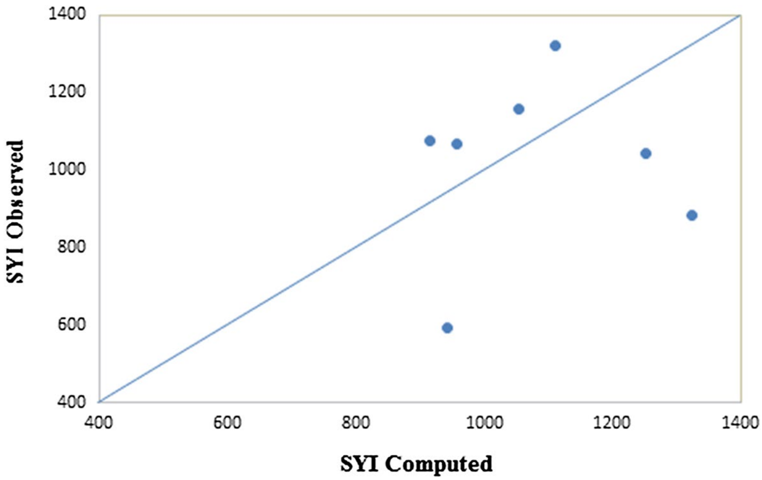

Fig. 8 Scatter plot between actual and predicted SYI by quadratic splines

an average value of APE = 20.99, ISE = 8.92, CE = − 199.90, CC = 8.95 and d = 0.15) was the 3rd best models.

As the literature review shows, cubic spline is popular because it is the lowest degree that allows separate control on the two end points and two end derivatives and it is also the lowest degree that allows inflection points (Prasad et al. 2018). DC power flow yields quick results at the expense of accuracy, whereas AC power flow compromises speed for accuracy. A solution to the above-mentioned problem is found by employing curve-fitting techniques especially the cubic spline interpolation technique (Othman et al. 2005). The cubic spline interpolation model (CSIM) method performs better than harmonic current injection model (HCIM) and harmonic voltage source model (HVSM) and provides a better fit for the voltage and current characteristics (Liu et al. 2010). The Niu et al. (2017) suggested that the conduction angle determined by using cubic splines shows significant match with

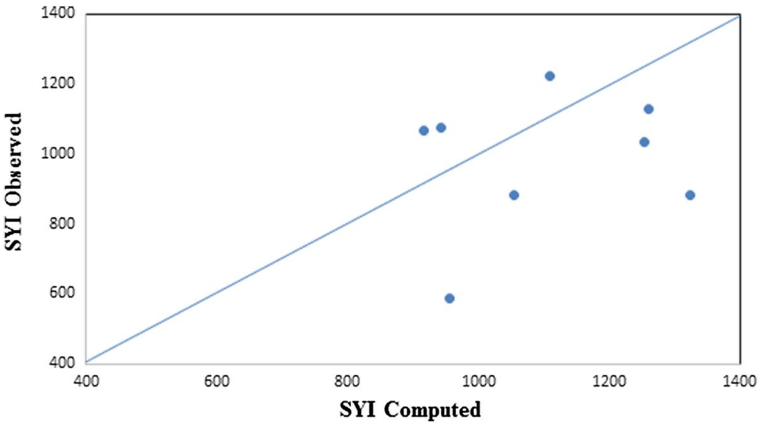

Fig. 9 Scatter plot between actual and predicted SYI by quintic splines

Table 5 Performance evaluation of splines

Spline	APE	ISE	CE	CC	d
Cubic spline	1.35	3.09	62.08	79.60	0.99
Quadratic spline	20.99	8.92	− 199.90	8.95	0.15
Quintic spline	19.59	7.84	− 165.73	19.30	0.26

the values obtained by simulation. Also, the use of cubic splines yields quicker results—a trait which would be beneficial for real-time applications.

5 Conclusion

In this study, an attempt has been made to develop a relationship between SYI and CN. Cubic, Quadratic and Quintic splines are developed between sediment yield index and curve number using eight sub-watershed data. This approximation is validated for sediment yield index for the remaining seven sub-watersheds. Subsequently, the SYI (observed) data of Mohgaon watershed exhibit a strong correlation with SYI derived using the cubic spline. High R^2 values (0.87) compared to quadratic spline ($R^2=0.40$) and quintic spline ($R^2=0.10$) support the general applicability of the proposed concept. The major output of this work is that on the basis of given set of curve numbers, we predict the approximate value of sediment yield index.

The sediment yield index is influenced by numerous physical attributes of a catchment. It changes with the slope, soil, land use land cover, drainage area and runoff-rainfall factors. This study establishes a link between SYI and CN. The spline approximation methods don’t consider the spatial variety of the numerous interfacing factors inside a watershed. The results of the present study need to be verified on a large data set covering watersheds from different climatic/geologic settings. The relationship between SYI and CN depends on the assumption that land use/land cover and other parameters remain constant with time. Therefore, incorporation of time dependency of these parameters in the GIS environment may be a scope for future study.If the value of curve number does not belong to the domain of approximation, then getting the approximation value of SYI is not possible. This problem may be resolved by applying certain techniques of extrapolation. It would be an interesting field of further extension of the present work.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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