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ORIGINAL PAPER

Assessing erosion prone areas in a watershed using interval rough-analytical hierarchy process (IR-AHP) and fuzzy logic (FL)

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Abstract

Soil erosion is one of the major land loss problems in agricultural land and is regarded as a serious environmental hazard worldwide. This study focused on watershed prioritization using morphometric parameters using Fuzzy Logic (FL), Interval Rough-Analytical Hierarchy Process (IR-AHP) and Geographic Information Systems (GIS) integration for Gusru Watershed, India. Fourteen morphometric parameters, including circulatory ratio (R_c), form factor (R_f), elongation ratio (R_e), compactness coefficient (C_c), drainage density (D_d), stream frequency (F_s), texture ratio (T), relief ratio (R_h), relative relief (R_r), ruggedness number (R_N), bifurcation ratio (R_b), average slope (S_a), length of overland flow (L_o), and hypsometric integral (HI) were evaluated to determine the erosion susceptibility. Each morphometric parameter was assigned a weight value by the FL and IR-AHP methods, and mapping and analysis were then carried out in the GIS environment. Our results showed that the sub-watersheds (SW) 9, 2, and 11 were most susceptible to soil erosion and the sub-watershed 1 was the least from the viewpoint of soil erosion ranking.

Keywords Watershed · Morphometric parameter · Soil erosion · AHP · Fuzzy logic

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1 Introduction

Soil erosion is an environmental, economic and social problem that affects all countries. For sustainable development of natural resources to diminish the impacts of natural calamities, a watershed could be taken as developmental unit (UNEP 1997). Although a number of factors are involved in soil erosion, a major agent is the water in the problem of land deterioration in most parts of the world. India's lands are not resistant to this type of natural hazards, since a total of 147 M ha soil loss were estimated in the country (Bhattacharyya et al. 2015).

Soil erosion, excess water flow or runoff, changes in river geometry, degradation of streams, sediment accumulation in river and stream characters are, to some extent, all water borne natural processes, which are related with morphometry (Meshram and Meshram, 2020). This clearly suggests that the morphometry of a basin is fundamental to the basin hydrology. Nowadays the latest technologies such as remote sensing (RS) and geographic information systems (GIS) have been so effectively utilized in the morphometric analyses as the old practices of measuring morphometry parameters were very time consuming and error prone (Gajbhiye et al. 2014a,b; Gajbhiye 2015a, b a,b; Meshram and Sharma 2017).

Nowadays, multi-criteria decision making (MCDM) techniques were introduced with various problem solutions in the complex decision making (Liu et al. 2006; Shih et al. 2007; Chang and Hsu 2009; Chang and Lin 2014; Salehi and Izadikhah 2014; Kobryń and Prystrom 2015; Mulliner et al. 2016; Mira et al. 2016; Malekian and Azarnivand 2016; Yu et al. 2017; Shojaie et al. 2017; Raju et al. 2017; Malekian and Azarniv 2016; Meshram et al. 2019, 2020a, b; Dahmardeh Ghaleno et al. 2020). Smithson (2012) divided the MCDM method into two categories: objective and subjective. In the former category, the natural distribution of the criteria is used to assess their effects on the study objective. Fuzzy logic (FL) is a commonly used method in the objective category (e.g., Ozelkanand Duckstein 2001; Yu et al. 2004; Suresh and Mujumdar 2004; Guan and Aral 2005; Yu and Chen 2005; Rao and Srinivas 2006; Chen and Chang 2010; Pourghasemi et al. 2012; Vahdani et al. 2013; John et al. 2014; Xu et al. 2016; Nguyen 2016; Danandeh Mehr et al. 2018).Jun et al. (2013) applied a fuzzy multi-criteria method to demarcate South Korea's flood susceptibility maps under the impacts of climate change. For Hamadan City, the basins were prioritized in accordance of flood intensity using FL technique (Sepehri et al., 2019). A fuzzy multi-criteria decision method to demarcate mapping of flood hazard zones of ungauged and data-scarce regions was developed by Kanani-Sadat et al. (2019) who considered each weight of every criterion in the fuzzy logic technique as a function of inter-criteria. Hence the multi-criteria methods with unique weights have many criteria due to limitations of inter-criteria dependency, it is then reasonable to ponder subjective and objective approaches together in the evaluation of criterion weights of (outer-criteria).

The method of analytical hierarchy process (AHP), introduced by Saaty (1980), is commonly used in natural hazard studies in the subjective category in order to determine the relative weights of parameters. Sepehri et al. (2017) implemented AHP in Gonbadchi area in Iran for flood hazard mapping. In the Middle East arid region, a demarcation of flood zones susceptibility using 10 criteria (namely, flow accumulation, distance to drainage network, elevation, LULC, annual rainfall, slope, geology, runoff, soil type, and drainage density) was evaluated by Mahmoud and Gan (2018). At this point it is worthwhile to remind that the task of flood risk mapping has been carried out using coupled physical hydrologic and hydraulic models (among others, Şen and Kahya, 2017). Moreover, the parameters have a weight as per their severity in analytical hierarchy. From the standpoint of erosion, Meshram et al. (2019) studied watershed priority ranking using AHP and Fuzzy Analytical hierarchy.

In mapping flood risk zones, Souissi et al. (2019) adopted MCDM-AHP using GIS tool using a set of different characteristic parameters (i.e., elevation, land use/-land cover, lithology, rainfall intensity, drainage density, distance from the drainage network, slope, and groundwater depth). Each criterion on relative weight was estimated on the basis of preference whose decision is made based on the AHP and other subjective approaches. Therefore, Gigović et al. (2017) and Pamucar et al. (2017) attempted to decrease this ambiguity considering Interval Rough Numbers (IRN). In identifying the best priority areas and landfill sites, Kharat et al. (2016) combined the following three methods: the fuzzy process, hierarchical approach and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS).

The quantitative relationships between soil erosion conditioning variables and their spatial distribution have been evaluated in GIS using fuzzy logic (Hembram and Saha 2018; Banerjee et al. 2018; Saha et al. 2019; Pilevar et al. 2020), machine learning approaches (Camilo et al. 2017; Rahmati et al. 2017; Al-Abadi et al. 2017; Conoscenti et al. 2018; Ghose and Samantaray 2019; Arabameri et al. 2019), the analytical hierarchy process (AHP) (Svoray et al. 2012; Arabameri et al. 2018a; Das et al. 2020).

Since in AHP and other subjective methods, the relative weight of the criteria is determined based on the decision maker's preferences, it causes high uncertainty in the final objective of the study. Therefore, in recent years, researchers have tried to reduce this uncertainty by using interval rough numbers (IRN) (Gigović et al. 2017; Pamucar et al. 2017). AHP offers an adaptable, low-cost, and understandable output for complex decision making (Saaty 1980).

This study aimed to make a soil susceptibility map of Gusru river watershed by integrating interval rough AHP (IR-AHP) and fuzzy logic and using GIS. The novelty of this work is that it uses IR-AHP and fuzzy logic algorithms with the morphometric parameter that are most consequential for soil erosion mapping or watershed prioritization. Soil erosion susceptibility mapping using IR-AHP and fuzzy logic has never been conducted in the Gusru River watershed; therefore, the results provide useful insights for planners and policymakers who desire to conserve and manage soil resources.

2 Materials and methods

2.1 Study area

The study domain (Gusru watershed) is located in the Madhya Pradesh state lying Satna Panna districts in India, and is bounded between 80°32′50.23′ E and 80°37′31.14′ E longitude, 24°6′32.75′ N and 24°16′24.07′ N latitude (Fig. 1). It occupies an area of 155 km^2 having elevation range from 339 to 628 m above the mean sea level. The Gusru River runs from east to west side and confluence with Tons river at Sagwania village. In the eastern part of the watershed, there is a small check dam, which primarily serves as an irrigation outlet. There is no other source of water for irrigation; as a result, rain-fed agriculture is of primary practice. The soil structure in the watershed is mainly comprised of sandy loam. Under rain-fed and irrigated conditions, the soils respond to a variety of crops and watershed management. Shale, sandstone and calcarious rocks are the dominant lithological units in the watershed. The study area descends from the plateau of Bhander and passes through the area between the escarpment of Bhander and the highlands of Kaimore.

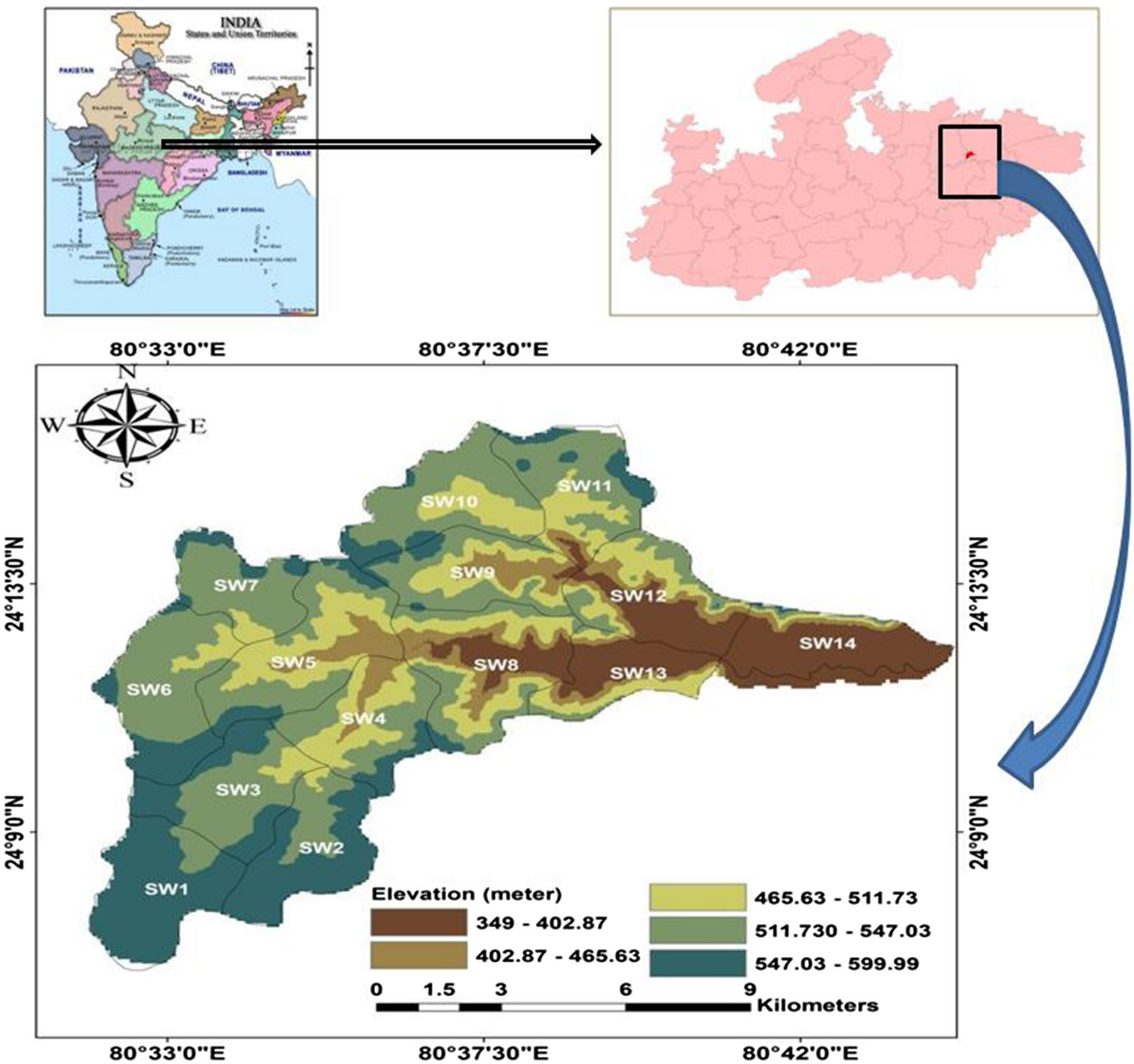

Fig. 1 Location map of the study area

2.2 Prioritization of watersheds using IR-AHP and fuzzy logic methods

A brief description of the fuzzy logic (FL) and interval rough analytical hierarchy (IR-AHP) methods for watershed prioritization will be presented in this section.

2.2.1 Fuzzy logic method

Zadeh (1965) introduced fuzzy theory in which fuzzy logic includes all ideas that use the fundamental principles of fuzzy sets or functions of membership. The membership function transfers the values of any parameters in range of [0, 1] to show the reliability level of parameters in obscure sets. The value of 0 implies that the desired value is not a member of the set under consideration whereas the value of 1 set refers to be a full member. On the basis of membership degree, the remaining values fall between 0 and 1.

A basic problem in the fuzzy logic is the lack of an optimal method in deciding how to sort in its parameters and membership function. The role of members is normally chosen on the basis of preference of decision-makers (Shahabi et al. 2015). Hence, the inter-weighing of all successful parameters appears to be more or less identical in different studies. In this study, the linear membership function (Eq. 1), one of the widely used membership functions in natural hazard studies (Ildoromi et al., 2019), was used to weight the parameters that have a direct relationship with soil erosion, whereas its inverse form (Eq. 2) was used to analyze for other criteria,which had an indirect relationship with the soil erosion.

$$\mathfrak{f}(x;a,b) = \left\{ \begin{array}{c} 0,\ \mathrm{x} \le a \\ \dfrac{x-a}{b-a}, a \le x \le b \\ 1, b \le x \end{array} \right\} \tag{1}$$

$$\mathfrak{f}^{-1}(x;b,a) = \mathfrak{f}(x;b,a) = \left\{ \begin{array}{c} 1, x \le a \\ 1 - \dfrac{x-a}{b-a}, a \le x \le b \\ 0, b \le x \end{array} \right\} \tag{2}$$

where x represents the specific value of the desired criteria and a and b show the minimum and maximum value of the desired criterion, respectively.

2.2.2 Interval rough numbers (IRN)

The uncertainty and errors in the data were quantified using the method of IRN-based rough numbers. A number of errors and subjectivities can accompany the group decision-making. In this regard, decision makers face a dilemma during assigning a certain value to criteria. It is then presumed that a numerical scale ranging from 1 to 9 must be graded as a function of the decision. Also, assume that there are three decision makers in the process of evaluating the criteria on erosion susceptibility (e.g., R_c). The first decision maker may assume that the initial importance of R_c criterion on soil erosion varies between 5 to 6 and the second decision maker may think that the significance of this criterion takes on a value between 6 and 7. Based on his experience, the third decision maker may consider a value in the range of 6 to 7. Then, we create attribute values to clarify the described uncertainty using operations on the rough numbers. Therefore, IRN ([5, 5.67], [6, 6.67]) will define the uncertainty of decision maker 1. Moreover, these values may be ([5.67, 6], [6.67, 7]) and ([5.67, 6], [6.67, 7]) for the decision makers 2 and 3, respectively (Pamucar et al. 2017).

Considering the novelty of the IRN methodology, there are few studies regarding the application of IRN in multi-criteria decision making. Because of its benefits, another aim in this study is set to promote the use of IRN in multi-criteria judgment-making.

Supposing that there is a set of k classes that represent decision maker (DM) preferences (in here are morphometric parameters), $\mathcal{R} = \left(J_1, J_2, J_3, \cdots J_k\right)$, in a situation that they fit in a series, which fulfills the conditions that $J_1 < J_2 < J_3 < \ldots\ldots < J_k$ and there is a another set of m classes that also stands as DM preferences, $\mathcal{R}^* = \left(I_1, I_2, I_3, \ldots\ldots I_k\right)$. All objects areexpressed in the world and linked with the DM preferences. In $\mathcal{R}^*$, each object class (e.g., R_c) has been shown in the interval $I_i = \left\{I_{li}, I_{ui}\right\}$, where the subsequent situation is fulfilled $I_{li} < I_{ui} (1 \le i \le m)$, and also $I_{li}, I_{ui} \in \mathcal{R}$. Then, I_{li} denotes the lower interval limit, while I_{ui} expresses the upper interval limit of the i$^{\text{th}}$ object class. If both the extremities of a class of objects (upper and lower limit) are listed in a way that is $I_{l1}^* < I_{l2}^* <, \ldots, I_{lj}^*, I_{u1}^* < I_{u2}^* <, \ldots, I_{uk}^* (1 \le j,\ k \le m)$, respectively, then, we can define two new sets containing the lower class of objects $\mathcal{R}_l^* = \left(I_{l1}^*, I_{l2}^*, I_{l3}^*, \ldots\ldots I_{lj}^*\right)$ and the upper class of objects $\mathcal{R}_u^* = \left(I_{u1}^*, I_{u2}^*, I_{u3}^*, \ldots\ldots I_{uk}^*\right)$, respectively. Then, for any class of objects $I_{li}^* \in \mathcal{R}\ (1 \le i \le j)$ and $I_{ui}^* \in \mathcal{R} (1 \le i \le k)$, we can define the lower approximations of I_{li}^* and I_{ui}^* as follows:

$$\underline{\mathrm{Apr}}(I_{li}^*) = \cup\left\{Y \in u/\mathcal{R}_l^*(Y) \le I_{li}^*\right\} \tag{3}$$

$$\underline{\mathrm{Apr}}(I_{ui}^*) = \cup\left\{Y \in u/\mathcal{R}_u^*(Y) \le I_{ui}^*\right\} \tag{4}$$

In the next equations, the upper approximations of I_{li}^* and I_{ui}^* are defined as:

$$\overline{\mathrm{Apr}}(\mathrm{I}_{\mathrm{li}}^{*}) = \cup\{\Upsilon \in \mathrm{u}/\mathcal{R}_{\mathrm{l}}^{*}(\Upsilon) \geq \mathrm{I}_{\mathrm{li}}^{*}\} \tag{5}$$

$$\overline{\mathrm{Apr}}(\mathrm{I}_{\mathrm{ui}}^{*}) = \cup\{\Upsilon \in \mathrm{u}/\mathcal{R}_{\mathrm{u}}^{*}(\Upsilon) \geq \mathrm{I}_{\mathrm{ui}}^{*}\} \tag{6}$$

Both the object classes (upper and lower I_{li}^{*} and I_{ui}^{*}) are expressed with their lower limits $\underline{\mathrm{Lim}}(I_{li}^{*})$ and $\underline{\mathrm{Lim}}(I_{ui}^{*})$ and upper limits $\overline{\mathrm{Lim}}(\mathrm{I}_{\mathrm{li}}^{*})$ and $\overline{\mathrm{Lim}}(\mathrm{I}_{\mathrm{ui}}^{*})$, respectively:

$$\underline{\mathrm{Lim}}(I_{li}^{*}) = \frac{1}{\mathcal{M}_{\mathcal{L}}}\sum \mathcal{R}_{l}^{*}(Y)|Y \in \underline{\mathrm{Apr}}(I_{li}^{*}) \tag{7}$$

$$\underline{\mathrm{Lim}}(I_{ui}^{*}) = \frac{1}{\mathcal{M}_{\mathcal{L}}^{*}}\sum \mathcal{R}_{u}^{*}(Y)|Y \in \underline{\mathrm{Apr}}(I_{ui}^{*}) \tag{8}$$

where $\mathcal{M}_{\mathcal{L}}$ and $\mathcal{M}_{\mathcal{L}}^{*}$ stands the sum of objects that are contained in the lower guess of aclass of objects I_{li}^{*} and I_{ui}^{*}, respectively. Upper limits $\overline{\mathrm{Lim}}(I_{li}^{*})$ and $\overline{\mathrm{Lim}}(I_{ui}^{*})$ are defined with Eqs. (7) and (8).

$$\overline{\mathrm{Lim}}(I_{li}^{*}) = \frac{1}{\mathcal{M}_{u}}\sum \mathcal{R}_{l}^{*}(Y)|Y \in \overline{\mathrm{Apr}}(I_{li}^{*}) \tag{9}$$

$$\underline{\mathrm{Lim}}(I_{ui}^{*}) = \frac{1}{\mathcal{M}_{u}^{*}}\sum \mathcal{R}_{u}^{*}(Y)|Y \in \overline{\mathrm{Apr}}(I_{ui}^{*}) \tag{10}$$

where $\mathcal{M}_{\mathrm{u}}$ and $\mathcal{M}_{\mathrm{u}}^{*}$ stands the sum of objects that are contained in the upper approximation of a class of objects I_{li}^{*} and I_{ui}^{*}, respectively.

The rough boundary interval for the lower object class I_{li}^{*} is represented as $\mathcal{RB}(I_{li}^{*})$ and indicates the interval between the lower and upper limits:

$$\mathcal{RB}(I_{li}^{*}) = \overline{\mathrm{Lim}}(I_{li}^{*}) - \underline{\mathrm{Lim}}(I_{li}^{*}) \tag{11}$$

While for the upper class of objects, the rough boundary interval I_{ui}^{*} is calculated as:

$$\mathcal{RB}(I_{ui}^{*}) = \overline{\mathrm{Lim}}(I_{ui}^{*}) - \underline{\mathrm{Lim}}(I_{ui}^{*}) \tag{12}$$

Then, uncertain object classes I_{li}^{*} and I_{ui}^{*} can be shown by their lower and upper limits.

$$\mathcal{RN}(I_{li}^{*}) = \overline{\mathrm{Lim}}(I_{li}^{*}), \underline{\mathrm{Lim}}(I_{li}^{*}) \tag{13}$$

$$\mathcal{RN}(I_{ui}^{*}) = \overline{\mathrm{Lim}}(I_{ui}^{*}), \underline{\mathrm{Lim}}(I_{ui}^{*}) \tag{14}$$

It was observed in each class of objects as distinct by its lower and upper limits, which makes interval rough number that is expressed as

$$\mathcal{IRN}(I_{i}^{*}) = [\mathcal{RN}(I_{li}^{*}), \mathcal{RN}(I_{ui}^{*})] \tag{15}$$

Special arithmetical operations that vary from arithmetic operations with classic rough numbers are defined by interval rough numbers.

2.2.3 IR-AHP mathematical model

Saaty (1980) evolved AHP, a method with numerous criteria frameworks for decision making. It was applied in several policy issues, remarkably in determining index weighting. This method provides the ability to measure the stability of decision makers' preferences in group decision making and allows manipulating quantitative and qualitative criteria. A final decision to use the AHP method could be based on the judgment of the decision maker. Hence, due to the subjectivity and ambiguity that occurs in group decision making, this study used the combination of IRN with the AHP method to exploit that mentality. This combination method can be summarized in five steps as follows (Gigović et al. 2017; Pamucar et al. 2017):

1. To organize a structure of hierarchical assessment criteria.In this step, the k experts provide the hierarchy of the difficultyin which the universal aim is located on the first level of the hierarchy and selected criteria are located in lower levels.
2. To fill the paired comparison matrix

   After providing a hierarchical structure of evaluation criteria, each expert compares the selected indices relative to each other as a format of comparison matrix (Z_{k}). In this context, the experts use the Saaty's 9-level linguistic scale (Table 1) to compare the indices.

$$Z_{k} = \begin{bmatrix} 1 & x_{12}^{e}; x_{12}^{e'} & \cdots & x_{1n}^{e}; x_{1n}^{e'} \\ x_{21}^{e}; x_{21}^{e'} & 1 & \cdots & x_{2n}^{e}; x_{2n}^{e'} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1}^{e}; x_{n1}^{e'} & x_{n2}^{e}; x_{n2}^{e'} & \cdots & 1 \end{bmatrix}_{n\times n} ; 1 \leq i, j \leq e \leq k \tag{16}$$

   In the above matrix, x_{ij}^{k} and $x_{ij}^{k'}$ represent the lower and upper limit of comparison matrix (each element of this matrix represents the comparison of criteria relative to each other (e.g., R_{c} relative to R_{f}). If there exists any uncertainty in expert decisions, they cannot choose a single value from the Saaty's 9-level linguistic scale, where the $x_{ij}^{k} \neq x_{ij}^{k'}$ are established and vice versa.
3. To establish the weight coefficients of the experts

   In subjective methods such as AHP, the consistency of experts' decision making needs to be evaluated. In the AHP method, Saaty (1980) introduced the following equation to calculate consistency rate (CR):

$$\mathcal{CR} = \mathcal{CI}/\mathcal{RI} \tag{17}$$

$$\mathcal{CI} = \frac{(\mathcal{I}_{\max} - n)}{(n-1)} \tag{18}$$

   where $\mathcal{CI}$: the consistency index;: the random index; $\mathcal{I}_{\max}$: the maximum eigen value of the comparison matrix; and n: the matrix rank. Such cases where a

Table 1 Saaty's 9-level linguistic scale

Preference factor	Degree of preference	Explanation
1	Equally	Two factors contribute equally to the objective
3	Moderately	Experience and judgment slightly to moderately favor one factor over another
5	Strongly	Experience and judgment strongly or essentially favor one factor over another
7	Very strongly	A factor is strongly favored over another and its dominance is showed in practice
9	Extremely	The evidence of favoring one factor over another is of the highest degree possible of an affirmation
2,4,6,8	Intermediate	Used to represent compromises between the preferences in weights1, 3, 5,7and 9
Reciprocals	Opposites	Used for inverse comparison

comparison matrix is acceptable are only the consistency rate being equal to or lesser than 0.1. In the case of the CR values being greater than 0.1, there exists an inconsistency in the comparison matrix, implying that a revision in expert decisions is needed.

4. To generate an averaged interval rough comparison matrix

The $\mathcal{Z}_k$ matrices are transferred to sequences matrices $\mathcal{X}^{*\mathcal{L}}$ and $\mathcal{X}^{*u}$ whose each element indicates the importance of index i relative to index j. Using Eqs. 1:14, the elements of $\mathcal{X}^{*\mathcal{L}}$ and $\mathcal{X}^{*u}$ are then transferred to $IRN(zij)$.

$$\begin{aligned}\mathcal{IRN}(\mathcal{Z}_{ij}) &= \left[\mathcal{RN}\left(\mathcal{Z}_{ij}^{\mathcal{L}}\right), RN(\mathcal{Z}_{ij}^{\prime u}\right] \\ &= \left[\mathcal{RN}\left(x_{ij}^{1\mathcal{L}}, x_{ij}^{2\mathcal{L}}, \ldots x_{ij}^{e\mathcal{L}}\right), \mathcal{RN}\left(x_{ij}^{1'u}, x_{ij}^{2'u}, \ldots x_{ij}^{e'u}\right)\right]\end{aligned} \tag{19}$$

$$\mathcal{RN}\left(\mathcal{Z}_{ij}^{\mathcal{L}}\right) = \mathcal{RN}\left(x_{ij}^{1\mathcal{L}}, x_{ij}^{2\mathcal{L}}, \ldots .x_{ij}^{e\mathcal{L}}\right) = \left\{\begin{array}{l} \mathcal{Z}_{ij}^{\mathcal{L}} = \frac{1}{m}\sum_{e=1}^{m} x_{ij}^{e\mathcal{L}} \\ \mathcal{Z}_{ij}^{u} = \frac{1}{m}\sum_{e=1}^{m} x_{ij}^{eu} \end{array}\right\} \tag{20}$$

$$\mathcal{RN}\left(\mathcal{Z}_{ij}^{\prime u}\right) = \mathcal{RN}\left(x_{ij}^{1'u}, x_{ij}^{2'u}, \ldots .x_{ij}^{e'u}\right) = \left\{\begin{array}{l} \mathcal{Z}_{ij}^{\mathcal{L}} = \frac{1}{m}\sum_{e=1}^{m} x_{ij}^{e'\mathcal{L}} \\ \mathcal{Z}_{ij}^{u} = \frac{1}{m}\sum_{e=1}^{m} x_{ij}^{e'u} \end{array}\right\} \tag{21}$$

where e is the number of experts (e = 1: m). In the pairs of evaluation parameters, the matrix $\mathcal{Z}$ expressing the average interval rough comparison matrix is estimated as follows:

$$Z = \begin{bmatrix} 1 & IRN(Z_{12}) & \cdots & IRN(Z_{1n}) \\ IRN(Z_{21}) & 1 & \cdots & IRN(Z_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ IRN(Z_{n1}) & IRN(Z_{n2}) & \cdots & 1 \end{bmatrix} \tag{22}$$

5. To calculate the priority criterion vector

For each n evaluation criterion, the priority criterion vector,$IRN(w_i)$, is calculated based on the following equations:

$$\begin{aligned}\mathcal{IRN}(w_{ij}) &= \left(\left[w_{ij}^{\mathcal{L}}, w_{ij}^{u}\right], \left[w_{ij}^{\prime\mathcal{L}}, w_{ij}^{\prime u}\right]\right) = \frac{\mathcal{IRN}(\mathcal{Z}_{ij})}{\sum_{j=1}^{n} \mathcal{IRN}(\mathcal{Z}_{ij})} \\ &= \frac{\left(\left[\mathcal{Z}_{ij}^{\mathcal{L}}, \mathcal{Z}_{ij}^{u}\right], \left[\mathcal{Z}_{ij}^{\prime\mathcal{L}}, \mathcal{Z}_{ij}^{\prime u}\right]\right)}{\left(\left[\sum_{j=1}^{n} \mathcal{Z}_{ij}^{\mathcal{L}}, \sum_{j=1}^{n} \mathcal{Z}_{ij}^{u}\right], \left[\sum_{j=1}^{n} \mathcal{Z}_{ij}^{\prime\mathcal{L}}, \sum_{j=1}^{n} \mathcal{Z}_{ij}^{\prime u}\right]\right)}\end{aligned} \tag{23}$$

Rough interval weight coefficients can be readily calculated as:

$$\mathcal{IRN}(w_i) = \left(\left[\sum_{j=1}^{n} w_{ij}^{\mathcal{L}}, \sum_{j=1}^{n} w_{ij}^{u}\right], \left[\sum_{j=1}^{n} w_{ij}^{\prime\mathcal{L}}, \sum_{j=1}^{n} w_{ij}^{\prime u}\right]\right)/n \tag{24}$$

2.2.4 Weighted overlay method (WOM)

After assigning the weights to criteria either by inter-criteria (i.e., fuzzy logic) or by outer-criteria (IR-AHP), WOM was used for combining these criteria and preparing soil erosion susceptibility maps. The replacement property is one of the most significant characteristics in this procedure (Raj and Shaji 2017; Thapa et al. 2017).Using this characteristic, another criterion that has a higher score will replace a criterion that has a lower score. The WOM method can be mathematically represented for integrating the parameters as follows:

$$\text{Final weight} = \sum x_{\mathrm{i}} w_{\mathrm{i}} \tag{25}$$

where w_i is the weight of inter-criterion of the i index, and x_{i} is the weight of outer-criterion of the i index.

2.3 Methodology

The first task is to set relevant morphometric criteria prior to soil erosion susceptibility mapping. Next, we first employed the fuzzy logic for the inter-criteria weighting, and then, the IR-AHP method for the outer-criteria weighting. Having the inter- and outer-criteria weight combination prepared, the soil erosion susceptibility map is finally graded into the areas of susceptibility to soil erosion. A flowchart of stages in our analysis is presented in Fig. 2.

2.3.1 Erodibility and mapping

Stream network is the basis of any morphometric study and their by prioritization of watershed digital elevation model (DEM) (with a resolution of 30 m) generated by Shuttle Radar Topography Mission (SRTM) data is a common tool to define a stream network and sub-watershed (SW) map. Different drainage network parameters (i.e., numbers & lengths) and watershed area, perimeter, width and length were determined in the GIS environment. In addition, the stream frequency, drainage density, circulatory ratio, form factor and elongation ratio were estimated using standard pertinent formulas. In order to do IR-AHP and fuzzy logic analysis, we have adopted the morphometric parameters for the 14 sub-watershed of Gusru watershed from the previous studies of Sharma et al. (2011).

In the first stage, we calculated morphometric parameters in each sub-watershed, and then compared. Then, the array of liking of sub-watersheds was estimated by the IR-AHP and fuzzy logic. Fourteen morphometric parameters were complied with respect to the sub-watershed scale for priority ranking in the study area.

2.3.2 Assigning weights and ranking criteria

In this analysis, the weights of parameters (inter-criteria and outer-criteria) were determined applying the fuzzy logic and IR-AHP methods based on knowledge-based approach. Scopes of the criteria which are dissimilar were eliminated using the fuzzy logic and the function and significance of the values of each criterion were specified in the sense of soil erosion. The IR-AHP, which is integrated with AHP approach, facilitates to eliminate the bias and ambiguity in the group judgment-making. This strategy may be perceived as a safer way in reducing the complexity of prioritizing watersheds.

3 Results and analysis

3.1 Morphometric analysis

A morphometric study should start with delineation of stream and their designation by any approach introduced

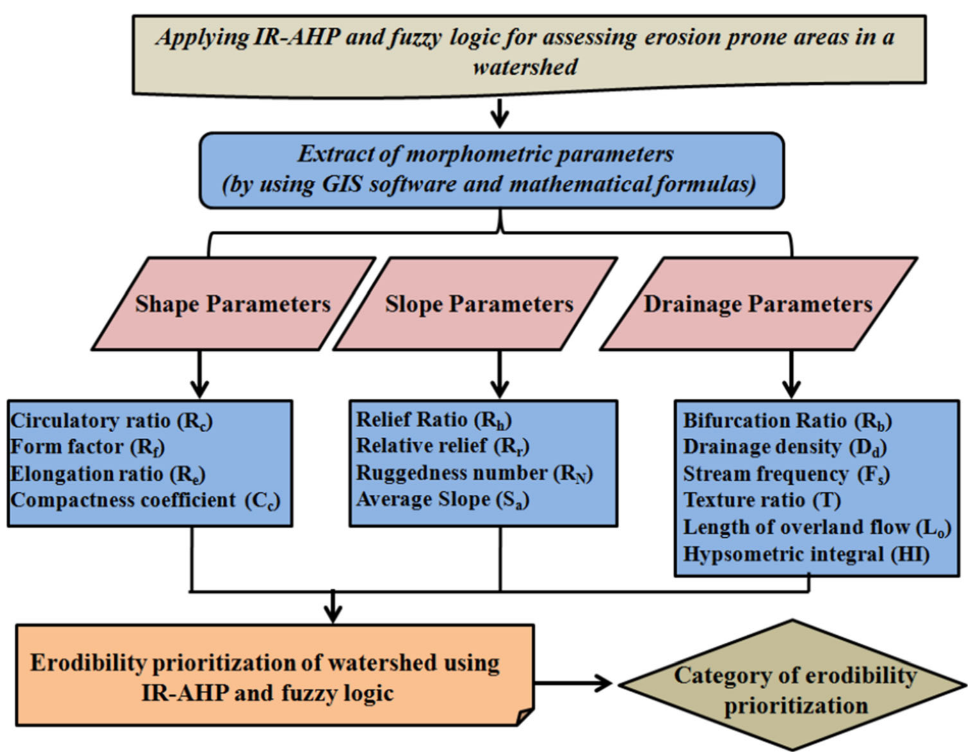

Fig. 2 Flowchart of the research

by Horton, Hargriv or Strahler. Therefore, stream ordering in our 14 sub-watersheds was referenced to the approach of Strahler (1964) in this study (Fig. 1).

3.2 Basic parameters

The watershed area, perimeter, width, stream number and length were all estimated using GIS technique and depicted in Fig. 3 and Table 2. It is evident that the SW 2 is the smallest one covering an area of 7.785 km^2 whereas the SW 8 has the largest area (14.723 km^2) among all the sub-watersheds. The length of watershed varies from 4.09 km (SW 7) to 5.91 km (SW 5).

3.3 Shape Parameter

Runoff characteristics (i.e., streamflow hydrograph) are influenced by the shape of watershed and can be evaluated through the circulatory ratio, form factor, and elongation ratio. These metrics of watershed revealed that the SWs 1, 6, 9, 10, 13, and 14 are elongated in shape, whereas the SWs 3, 4, 5, 7, and 11 are less elongated, and the SWs 1, 8 and 12 have an oval shape (Table 2). In the elongated basin hydrograph of stream flow or discharge takes smooth shape which explains that the longer time will be taken by water from most remote point of watershed to its outlet. In case of oval and less elongated basins water comes to outlet very fast and less time as compared to elongated basins which causes excessive peak value. For SW 12 and 14 compactness coefficient values are 1.149 and 1.430 respectively, which explains that SW 14 is more compact then SW 12.

3.4 Drainage Parameters

Among the linear parameters, the drainage density is very important parameter and has a relative links with stream length and watershed area. Figure 3 and Table 2 show drainage densities of sub-watersheds in Gusru watershed in which SW 14 possesses the highest value (D_d = 4.994 km/km^2) whereas SW 6 has the lowest drainage density value (D_d = 2.454 km/km^2). The majority of sub-watersheds 1–5 and 7–13 have the D_d value in the same ranges. Sub-watershed 6 has permeable conditions of sub soil strata indicated by its low D_d value. Among the sub-watersheds of the study area the sub-watershed 14 having the highest D_d

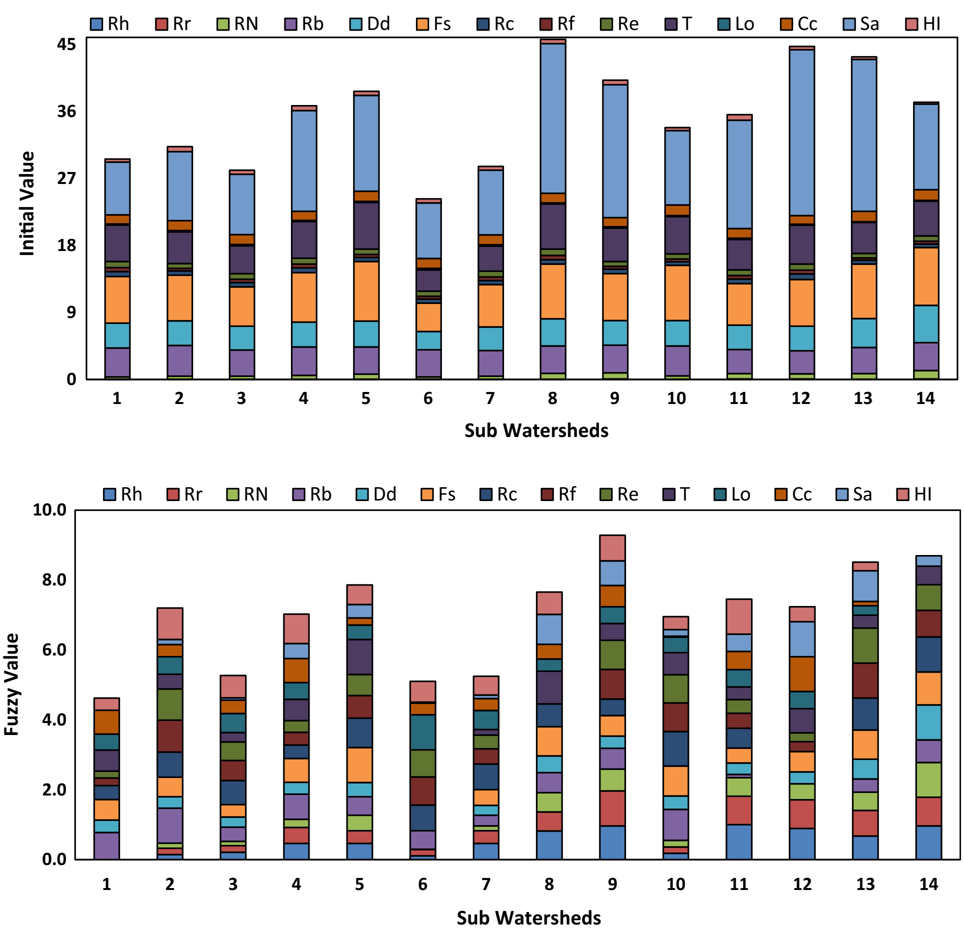

Fig. 3 Initial value combination of morphometric parameters and their fuzzy values

Table 2 Sub-watershed wise morphometric parameters

Sub-watershed	R_h	R_r	R_N	R_b	D_d	F_s	R_c	R_f	R_e	T	L_o	C_c	S_a	HI
1	0.019	0.006	0.304	3.889	3.372	6.264	0.651	0.530	0.822	4.902	0.148	1.239	7.089	0.410
2	0.023	0.008	0.425	4.115	3.293	6.165	0.564	0.340	0.658	4.275	0.152	1.331	9.275	0.700
3	0.025	0.008	0.409	3.521	3.199	5.299	0.573	0.433	0.743	3.776	0.156	1.321	8.121	0.560
4	0.032	0.011	0.499	3.833	3.328	6.663	0.654	0.490	0.790	4.928	0.15	1.236	13.524	0.670
5	0.032	0.010	0.670	3.646	3.488	8.016	0.531	0.414	0.726	6.270	0.143	1.372	12.89	0.520
6	0.022	0.008	0.312	3.643	2.454	3.817	0.56	0.370	0.686	2.859	0.204	1.335	7.467	0.540
7	0.032	0.010	0.420	3.417	3.180	5.700	0.561	0.472	0.775	3.385	0.157	1.335	8.680	0.510
8	0.042	0.012	0.763	3.681	3.670	7.335	0.582	0.591	0.868	6.058	0.136	1.311	20.115	0.560
9	0.046	0.017	0.827	3.705	3.334	6.284	0.631	0.356	0.673	4.495	0.15	1.259	17.845	0.610
10	0.024	0.008	0.462	4.005	3.421	7.426	0.494	0.363	0.680	5.013	0.146	1.422	9.998	0.420
11	0.047	0.015	0.742	3.208	3.285	5.598	0.606	0.473	0.776	4.092	0.152	1.284	14.566	0.750
12	0.044	0.015	0.684	3.113	3.319	6.268	0.758	0.513	0.809	5.217	0.151	1.149	22.295	0.450
13	0.038	0.014	0.737	3.495	3.899	7.322	0.513	0.315	0.634	4.130	0.128	1.395	20.416	0.360
14	0.046	0.015	1.134	3.759	4.994	7.785	0.489	0.381	0.696	4.671	0.1	1.430	11.553	0.230

value which means this watershed has well developed stream network. This well developed stream network shows high elevation, scattered vegetation and substrata have fragile material in that watershed (Nautiyal 1994).

SW 6 & 5 has stream frequency 3.817 and 8.016 number/km which shows the watershed with maximum stream frequency has well developed stream network and more important implication on severe soil erosion in the watershed. The intensity of erosion in a watershed increases with the stream frequency, which is positively correlated with the drainage density. Texture ratio (T) of all sub-watersheds ranged from 2.859 (in the SW 6) to 6.270 (in the SW 5).

3.5 Slope parameters

The difference of elevation in most remote point of watershed and its outlet is known as total relief (H) that ranged between 90 and 227 m in all the sub-watersheds of the Gusru watershed. Rate of runoff water and sediment volume depends on the relief ratio as this parameter provides the potential energy to these factors in the study area its range varies from 0.019 to 0.047 (SW 1 & 11). The range of relative relief (R_r) was between 0.006 (SW 1) and 0.017 (SW 9). Ruggedness number (R_N) was estimated as 0.304 for sub-watershed 1as being the lowest value and 1.134 for SW 14 as being the highest value in this study. More R_N value shows the more roughness of watershed and as a consequence more soil loss. Slope range in Gusru river watersheds was observed between 7.089% (SW 1) and 20.416% (SW 13). Sub-watersheds 2, 4, 9, and 11 are inequilibrium/youthful stage, and sub-watersheds 1, 3, 5, 6, 7, 8, 10, 12, and 13 are in equilibrium/mature stage, and finally sub-watershed 14 in monodnock stage.

3.6 Erodibility criteria for sub-watershed prioritization by fuzzy logic and IR-AHP models

The main features of this approach are its consistency and applicability with an emphasis on watershed prioritization criteria. The MCDM method is often used in ranking, accepting, rejecting, and evaluating the number of optimum choices. As each being an MCDM method in weighting criteria procedure, both IR-AHP and fuzzy logic methods were employed in this study.

3.6.1 Inter-criteria weighting (Fuzzy logic)

The form factor (R_f), which is defined by dividing the sub-basin area by the longest length of drainage network has an inverse relationship with soil erosion in such a way that the phenomenon of sedimentation dominates soil erosion in long-narrow watersheds. The value of R_f in all sub-watersheds, which varies between 0.591 (SW 8) and 0.315 (SW 13), was therefore transferred between 0 (SW 8) and 1 (SW 13). The relief ratio (R_e) like R_f has an inverse relationship with soil erosion. For small values of R_f, the case study has a high relief and steep ground slopes and therefore it has more susceptibility to soil erosion. Using Eq. 2, the values of 0 and 1 were assigned to SW 8 (0.868) and SW 13 (0.634), which are the maximum and minimum values of R_e, respectively. Once the parameters of circulatory ratio and compactness coefficient (R_c and C_c) are related to watershed penetration capacity, they are negatively correlated with soil erosion. In this context, Eq. 2 was used to assign inter-criteria weighting to R_c and C_c.

Bifurcation ratio (R_b) having a relation with the watershed structure complexity is affected by geological, climate, topographical, and ecological watershed factors. This ratio for sub-watersheds changes between 3.113 (SW 12) and 4.113 (SW 2). As reported by Meshram et al. (2020a,b) some parameters like average slope of watershed, bifurcation ratio, hypsometric integral, and ruggedness number, length of overland flow, relative relief, relief ratio, drainage density, stream frequency and texture ratio have a direct relationship with soil erosion. Therefore, Eq. 1 has been utilized to assign inter-criteria weighting to this index (Fig. 3). Drainage density (D_d) is a representation for vegetation cover and relief rate. In areas with lower D_d, there is good vegetation cover and low relief and vice versa. In our study, SW 14 and 6 were found to have the maximum and minimum rates of D_d, respectively. Equation 1 was once again used in identifying inter-criteria weighting (Fig. 3).The length of overland flow (L_o) is also a vital indices affecting physiographic and hydrological features of the watershed. The higher value of this index is related to mild slopes and vice versa. SW 6 has a maximum value (0.204) of L_o, being most susceptible to erosion, whereas sub-watershed 14 (1) has a minimum value of L_o, being least susceptible to soil erosion. Here, Eq. 1 was used to determine the inter-criteria of L_o (Fig. 3).

One of the important geometric indices which are used to determine the erosion potential of streams is related to ruggedness number (R_N). This index has a positive correlation with soil erosion. Equation 1 was used to determine the inter-criteria of R_N (Fig. 3).The relief ratio and relative relief (R_h and R_r) are main hydrological indices which are dependent on the slope of drainage network. In this study, sub-watersheds 9 and 1 have the maximum and minimum values of R_h and R_r, respectively. Since these indices have a direct relationship with soil erosion, Eq. 1 was used to transfer their values to range of 0 to 1 (Fig. 3). Stream frequency, average slope, and hypsometric integral parameters (F_s, S_a, and HI) represent the roughness coefficient in a watershed and higher values of these indices indicate that the watershed has low permeability. As these parameters have a positive relationship with soil erosion and, in this study, their inter-criteria weighting was assigned by Eq. 1 (Fig. 3).

In the case study, a scale ranging from 1 to 14 was used for assigning a primary score to the 14 morphometric parameter based on their role on soil erosion. The score 1 means that the lowest impact on soil erosion and number 14 means the most impact on soil erosion. a map was generated in which the higher the values, the stronger their effect is on soil erosion, and vice versa (Fig. 3). Finally, Eq. (1) was used for inter-weighting this criterion.

3.6.2 Outer-criteria weighting (IR-AHP)

The inter-criteria weighting was used to remove the dimensions of indices and assign initial weights to values of each index without considering other indices. It is obvious that each index relative to other indices has a different role and importance in the degree of erosion. To do this, three experts were selected to specify the relative importance of indices in the format of comparison matrix (Table 3) whose values bring about uncertainty in experts' decisions. For example, the expert is in a dilemma to choose the values between 3 and 4 in the comparison matrix of expert 1 in array R_c-R_b (Table 4). Having acquired the comparison matrixes, the next step is to calculate the final consistency ratio of each matrix, which is the average of consistency ratio (CR) of lower (CR^e) and upper ($CR^{e'}$) limit of that matrix (Table 4). Since the final value of consistency ratio for each expert is lower than 0.1, they can be used to determine the final weights of erosion-related indices. Using Eqs. 3–15, the comparison matrices (Z_k) (Table 5) were transferred to averaged interval rough comparison (Table 5). After calculating $IRN(zij)$, the priority criteria vector (Table 6) and rough interval weights coefficients (Fig. 4) were calculated through Eqs. 23 and 24.

Following the assessment of weights of criteria by inter-criteria (i.e. fuzzy logic) and outer-criteria (IR-AHP), weighted overlay method (WOM) was used for a combination of these criteria and to prepare the soil erosion susceptibility map (Fig. 5).

4 Discussions

The employed methodology's main characteristics are its simplicity and adaptability, with an emphasis on the criteria for controlling water routing when the peak exceeds the drainage network capacity. MCDM is commonly used for ranking, accepting, rejecting, and finding the number of ideal options (Fernández and Lutz 2010). IRAHP and fuzzy logic were utilized as two MCDA approaches for criteria weighting in this study.

According to the literature review, no unique MCDM model is superior to others in all circumstances, and the performance of different MCDM models varies depending on the condition of each hydrological system. As a result, it has been tested and shown that using an MCDM method to combine outputs (from different models) can lead to more accurate findings (Meshram et al. 2021). The concept of an MCDM model like this has already been applied in environmental (Soltani et al. 2015; Zavadskas et al. 2015), quality management (Lupo, 2015), GIS (Latinopoulos and Kechagia, 2015), safety and risk management

Table 3 Pairwise comparison matrices of experts

	R_h	R_r	R_N	R_b	D_d	F_s	R_c	R_f	R_e	T	L_o	C_c	S_a	HI
Expert 1														
R_h	(1,1)													
R_r	(1,2)	(1,1)												
R_N	(3,3)	(1,2)	(1,1)											
R_b	(3,4)	(2,3)	(2,3)	(1,1)										
D_d	(4,5)	(3,4)	(3,4)	(2,3)	(1,1)									
F_s	(3,4)	(4,5)	(1,2)	(1,2)	(1,2)	(1,1)								
R_c	(4,5)	(3,4)	(2,3)	(3,4)	(2,3)	(2,3)	(1,1)							
R_f	(3,4)	(4,5)	(4,5)	(3,4)	(3,4)	(1,2)	(2,3)	(1,1)						
R_e	(1,2)	(3,4)	(2,3)	(2,3)	(2,3)	(2,3)	(1,2)	(2,3)	(1,1)					
T	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,2)	(1,1)	(2,2)	(2,2)	(1,1)				
L_o	(4,5)	(3,4)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(1,1)			
C_c	(3,4)	(4,5)	(3,4)	(1,2)	(3,4)	(1,2)	(2,3)	(2,3)	(2,3)	(2,3)	(3,4)	(1,1)		
S_a	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,3)	(2,2)	(1,2)	(1,2)	(1,2)	(1,3)	(1,3)	(1,1)	
HI	(6,7)	(5,6)	(6,7)	(4,5)	(3,4)	(6,7)	(6,7)	(5,6)	(6,7)	(7,8)	(5,6)	(5,6)	(6,7)	(1,1)
Expert 3														
R_h	(1,1)													
R_r	(1,1)	(1,1)												
R_N	(3,5)	(1,5)	(1,1)											
R_b	(3,4)	(2,2)	(1,2)	(1,1)										
D_d	(4,5)	(3,3)	(2,3)	(2,2)	(1,1)									
F_s	(3,4)	(4,4)	(1,1)	(1,1)	(1,1)	(1,1)								
R_c	(4,5)	(3,3)	(2,2)	(3,4)	(2,2)	(2,2)	(1,1)							
R_f	(3,4)	(4,4)	(4,5)	(3,3)	(3,4)	(1,1)	(2,2)	(1,1)						
R_e	(1,1)	(3,4)	(2,2)	(2,2)	(2,3)	(2,2)	(1,1)	(2,3)	(1,1)					
T	(1,1)	(1,1)	(1,1)	(1,2)	(1,1)	(1,1)	(1,1)	(2,3)	(2,2)	(1,1)				
L_o	(4,5)	(3,3)	(2,2)	(2,2)	(2,3)	(2,2)	(2,2)	(2,2)	(2,2)	(2,3)	(1,1)			
C_c	(3,5)	(4,4)	(3,4)	(1,1)	(3,3)	(1,1)	(2,2)	(2,2)	(2,2)	(2,2)	(3,3)	(1,1)		
S_a	(2,3)	(2,2)	(2,2)	(2,3)	(2,2)	(2,2)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	
HI	(6,6)	(5,5)	(6,7)	(4,5)	(3,4)	(6,6)	(6,7)	(55,)	(6,6)	(7,8)	(5,6)	(5,5)	(6,7)	(1,1)

Table 4 CR for comparison matrices

Expert	CR^e	$CR^{e'}$	CR
1	0.067	0.099	0.083
2	0.068	0.096	0.082
3	0.097	0.099	0.098

(Ilangkumaran et al. 2015), operation research and soft computing (Angilella and Mazzù, 2015; Zhu et al. 2015; Chen, 2015; Roszkowska and Wachowicz, 2015). As a result, it is advised that a fuzzy technique be proposed in order to increase the accuracy of the prediction MCDM models. The results of fuzzy approaches were shown to be more accurate than those of an MCDM model.

Several studies have used fuzzy sets for soil erosion studies (Al-Abadi et al. 2017; Saha et al. 2019; Pilevar et al. 2020). The fuzzy set method has several advantages, including (i) converting all data to a range of 0–1 using fuzzy membership functions as an excellent mechanism for solving different magnitudes at different data layers, and (ii) accurately assessing soil erosion for watershed prioritization using a different morphometric parameter.

The difference in soil erosion sensitive zones between this study and other studies (Le Cozannet et al. 2013; Pradeep et al. 2014; Vijith and Dodge-Wan 2019; Meshram et al. 2020a,b; Alvandi et al. 2021) is one of the most significant differences. Some studies, for example, may over- or under-estimation soil susceptibility based on their criterion weighting approach, resulting in erroneous soil erosion susceptible mapping precision.

Table 5 Interval rough average matrix

	R_h	R_r	R_N	C_c	S_a	HI
R_h	([1,1],[1,1])	([0.45,0.78],[0.61,0.88])	([0.22,0.29],[0.33,0.33])	([0.2,0.22],[0.28,0.32])	([0.28,0.32],[0.4,0.48])	([0.13,0.15],[0.15,0.16])
R_r	([1.11,1.55],[1.5,2.5])	([1,1],[1,1])	([0.27,0.42],[0.72,0.94])	([0.18,0.22],[0.22,0.24])	([0.3,0.42],[0.4,0.48])	([0.15,0.18],[0.18,0.19])
R_N	([3,3],[3.5,4.5])	([1.11,1.55],[2.6,4.1])	([1,1],[1,1])	([0.22,0.24],[0.28,0.32])	([0.3,0.42],[0.4,0.48])	([0.13,0.14],[0.15,0.16])
C_c	([3.11,3.55],[4.44,4.88])	([4.11,4.55],[4.5,5.5])	([3.11,3.55],[4.11,4.55])	([1,1],[1,1])	([0.35,0.73],[0.62,0.92])	([0.15,0.18],[018,0.19])
S_a	([2.11,2.55],[3.11,3.55])	([2.11,2.55],[2.5,3.5])	([2.11,2.55],[2.5,3.5])	([1.22,2.11],[1.8,3.3])	([1,1],[1,1])	([0.13,0.15],[0.15,0.16])
HI	([6.11,6.55],[6.5,7.5])	([5.11,5.55],[5.5,6.5])	([6.11,6.55],[7.111,7.55])	([5.11,5.55],[5.5,6.5])	([6.11,6.55],[7.11,7.55])	([1,1],[1,1])

Table 6 Priority criteria vector

	R_h	R_r	R_N	C_c	S_a	HI
R_h	([0.14,0.17],[0.17,0.22])	([0.1,0.1],[0.10,0.13])	([0.04,0.05],[0.05,0.05])	([0.04,0.04],[0.047,0.049])	([0.05,0.06],[0.07,0.07])	([0.02,0.02],[0.02,0.03])
R_r	([0.21,0.23],[0.22,0.29])	([0.11,0.14],[0.15,0.19])	([0.05,0.06],[0.05,0.06])	([0.02,0.03],[0.03,0.03])	([0.05,0.06],[0.05,0.06])	([0.02,0.02],[0.02,0.03])
R_N	([0.3,0.3],[0.3,0.36])	([0.13,0.15],[0.22,0.27])	([0.06,0.08],[0.1,0.12])	([0.02,0.02],[0.02,0.02])	([0.03,0.03],[0.03,0.04])	([0.01,0.01],[0.01,0.01])
C_c	([0.1,0.1],[0.1,0.12])	([0.12,0.012],[0.13,0.14])	([0.1,0.1],[0.1,0.11])	([0.02,0.02],[0.02,0.03])	([0.01,0.02],[0.01,0.02])	([0.004,0.005],[0.005,0.005])
S_a	([0.09,0.1],[0.09,0.11])	([0.08,0.09], [0.09,0.1])	([0.08,0.09],[0.09,0.1])	([0.05,0.07],[0.06,0.08])	([0.02,0.03],[0.03,0.04])	([0.004,0.005],[0.005,0.006])
HI	([0.07,0.08],[0.08,0.08])	([0.06,0.07],[0.07,0.07])	([0.08,0.08],[0.08,0.08])	([0.06,0.07],[0.07,0.07])	([0.07,0.08],[0.08,0.08])	([0.01,0.01],[0.01,0.01])

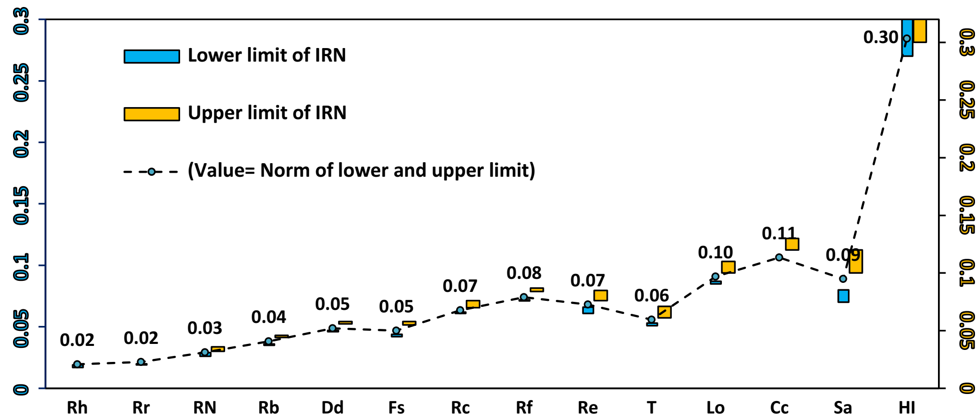

Fig. 4 Rough interval weights coefficients

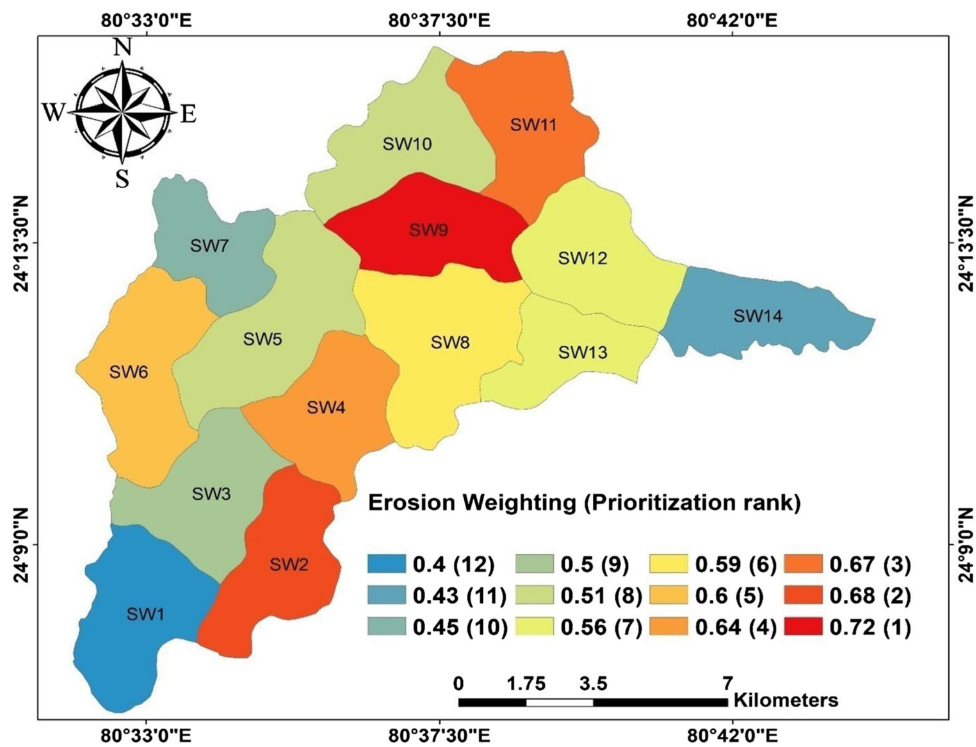

Fig. 5 Soil erosion susceptibility map

Because the use of subjective approaches (AHP) alone for soil erosion mapping has some limits in terms of weighting indices, it is important to divide the criterion into subclasses and then assign initial weights to these subclasses. This reduces the variability of spatial distribution of criteria, resulting in rigid prioritization maps. The integration of fuzzy logic and the IR-AHP approach as objective and subjective methodologies was utilized to solve this problem in this study.

5 Conclusion

Soil erosion is one of the major land loss problems in agricultural lands and is regarded as a serious environmental hazard worldwide. In the present study, fourteen most effective morphometric parameters were selected in soil erosion evaluation. These parameters were weighted on the basis of their effects on erosion using subjective (IR-AHP) and objective (fuzzy logic) methods. Finally, the weighted criteria were overlaid to have soil erosion map of the study area. The following conclusions can be drawn from this study:

1 Sub-watersheds 14, 2, and 11 have most susceptibility to soil erosion.

2 Sub-watershed 1has least susceptibility to soil erosion. It was shown that the integration of AHP- Rough and fuzzy logic can be a powerful tool to investigate soil erosion.

The interval rough approach is valid in other areas of MCDM. Further implementation of the interval rough method in conjunction with established decision-making models would significantly reduce the complexity and subjectivity prevalent in decision-making, in particular in group judgment-making.

The drainage network is most important geomorphology feature of watershed which effect on soil erosion. Therefore, the accurate evaluation of drainage network feature has a main role on the accuracy of the desired output. In the current study, the main concentration of authors was on the simple properties of drainage network i.e. length of drainage network or the number of it. Therefore, in future studies in order to have more accuracy of soil erosion mapping, it is necessity to consider drainage network irregularity property such as Fractality or entropy of drainage network. Regarding using methodology, it can be said that the core of the used methodology in the current study was AHP. One the main weakness of the AHP is the lots number of paired comparisons, causing uncertainty in produced desired output such as soil erosion mapping. Therefore, it is suggested that in the future, the authors used new methods which reduce the number of paired comparisons such as Best Worst Method (BWM) or Full Consistency Method (FUCOM) as two the newest subjectively MCDM methods which dramatically reduce the number of paired comparisons and the uncertainty in the produced maps.

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Data availability The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no competing interests.

References

Al-Abadi AM, Shahid S, Ghalib HB, Handhal AM (2017) A GIS-based integrated fuzzy logic and analytic hierarchy process model for assessing water-harvesting zones in Northeastern Maysan Governorate, Iraq. Arab J Sci Eng 42:2487–2499. https://doi.org/10.1007/s13369-017-2487-1

Alvandi E, Soleimani-Sardo M, Meshram SG, Giglou BF, Dahmardeh Ghaleno MR (2021) Using Improved TOPSIS and Best Worst Method in prioritizing management scenarios for the watershed management in arid and semi-arid environments. Soft Comput 25:11363–11375. https://doi.org/10.1007/s00500-021-05933-9

Arabameri A, Rezaei K, Pourghasemi HR, Lee S, Yamani M (2018) GIS-based gully erosion susceptibility mapping: a comparison among three data-driven models and AHP knowledge-based technique. Environ Earth Sci 77:628. https://doi.org/10.1007/s12665-018-7808-5

Arabameri A, Pradhan B, Lombardo L (2019) Comparative assessment using boosted regression trees, binary logistic regression, frequency ratio and numerical risk factor for gully erosion susceptibility modelling. CATENA 183: https://doi.org/10.1016/j.catena.2019.104223

Angilella S, Mazzù S (2015) The financing of innovative SMEs: a multi-criteria credit rating model. Eur J Oper Res 244:540–554. https://doi.org/10.1016/j.ejor.2015.01.033

Banerjee R, Srivastava P, Pike A, Petropoulos G (2018) Identification of Painted Rock-Shelter Sites Using GIS Integrated with a Decision Support System and Fuzzy Logic. ISPRS Int J Geo-Info 7(8):326. https://doi.org/10.3390/ijgi7080326

Bhattacharyya R, Ghosh BN, Mishra PK, Mandal B, Rao CS, Sarkar D, Das K, Anil KS, Lalitha M, Hati KM, Franzluebbers AJ (2015) Soil Degradation in India: challenges and potential solutions. Sustainability 7(4):3528–3570. https://doi.org/10.3390/su7043528

Camilo DC, Lombardo L, Mai PM, Dou J, Huser R (2017) Handling high predictor dimensionality in slope-unitbased landslide susceptibility models through LASSO-penalized Generalized Linear Model. EnvironModell Soft 97:145–156. https://doi.org/10.1016/j.envsoft.2017.08.003

Chang CL, Hsu CH (2009) Multi-criteria analysis via the VIKOR method for prioritizing land-use restraint strategies in the Tseng-wen reservoir watershed. J Environ Manag 90:3226–3230

Chang CL, Lin YT (2014) Using the VIKOR method to evaluate the design of a water quality monitoring network in a watershed. Intl J Environ Sci Technol 11:303–310

Chen HW, Chang NB (2010) Using fuzzy operators to address the complexity in decision making of water resources redistribution in two neighboring river basins. Adv Water Res 33:652–666. https://doi.org/10.1016/j.advwatres.2010.03.007

Chen TY (2015) The inclusion-based TOPSIS method with interval-valued intuitionistic fuzzy sets for multiple criteria group decision making. Appl Softw Comput 26:57–73

Conoscenti C, Agnesi V, Cama M, Caraballo-Arias NA, Rotigliano E (2018) Assessment of gully erosion susceptibility using multivariate adaptive regression splines and accounting for terrain connectivity. Land DegradDev 29:724–736. https://doi.org/10.1002/ldr.2772

Dahmardeh Ghaleno MR, Meshram SG, Alvandi E (2020) Pragmatic approach for prioritization of flood and sedimentation hazard potential of watersheds. Soft Comput. https://doi.org/10.1007/s00500-020-04899-4

Danandeh Mehr A, Nourani V, Kahya E, Hrnjica B, Sattar AMA, Yaseen ZM (2018) Genetic programming in water resources engineering: a state-of-the-art review. J Hydrol 566:643–667. https://doi.org/10.1016/j.jhydrol.2018.02.043

Das B, Bordoloi R, Thungon LT et al (2010) An integrated approach of GIS, RUSLE and AHP to model soil erosion in West Kameng watershed. Arunachal Pradesh. J Earth Syst Sci 129:94. https://doi.org/10.1007/s12040-020-1356-6

Fernández DS, Lutz MA (2010) Urban flood hazard zoning in Tucumán Province. Argentina, using GIS and multicriteria decision analysis. 111(1–4):90–98. https://doi.org/10.1016/j.enggeo.2009.12.006

Gajbhiye S, Mishra SK, Pandey A (2014a) Prioritizing erosion-prone area through morphometric analysis: an RS and GIS perspective. Appl Water Sci 4(1):51–61

Gajbhiye S, Mishra SK, Pandey A (2014b) Hypsometric analysis of Shakkar river catchment through geographical information system. J GeolSoc India 84:192–196. https://doi.org/10.1007/s12594-014-0123-1

Gajbhiye S (2015a) Morphometric analysis of a Shakkar river catchment using RS and GIS. Int J U- E-Ser, SciTechnol 8(2):11–24. https://doi.org/10.14257/ijunesst.2015.8.2.02

Gajbhiye S (2015b) Estimation of surface runoff using remote sensing and geographical information system. Int J U- E-Serv, SciTechnol 8(4):118–122. https://doi.org/10.14257/ijunesst.2015.8.4.12

Gigović L, Pamučar D, Bajić Z, Drobnjak S (2017) Application of GIS-interval rough AHP methodology for flood hazard mapping in urban areas. Water 9(6):360. https://doi.org/10.3390/w9060360

Ghose DK, Samantaray S (2019) Sedimentation Process and Its Assessment Through Integrated Sensor Networks and Machine Learning Process. In: Mishra B, Dehuri S, Panigrahi B, Nayak A, Mishra B, Das H (eds) Computational Intelligence in Sensor Networks. Studies in Computational Intelligence, vol 776. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-662-57277-1_20

Guan J, Aral MM (2005) Remediation system design with multiple uncertain parameters using fuzzy sets and genetic algorithm. J Hydrol Eng 10(5):386–394. https://doi.org/10.1061/(ASCE)1084-0699(2005)10:5(386)

Hembram TK, Saha S (2018) Prioritization of sub-watersheds for soil erosion based on morphometric attributes using fuzzy AHP and compound factor in Jainti River basin, Jharkhand, eastern India. Environ Dev Sustain. https://doi.org/10.1007/s10668-018-0247-3

Ilangkumaran M, Karthikeyan M, Ramachandran T, Boopathiraja M, Kirubakaran B (2015) Risk analysis and warning rate of hot environment for foundry industry using hybrid MCDM technique. Saf Sci 72:133–143. https://doi.org/10.1016/j.ssci.2014.08.011

Jun KS, Chung ES, Kim YG, Kim Y (2013) A fuzzy multi-criteria approach to flood risk vulnerability in South Korea by considering climate change impacts. Expert Syst Appl 40:1003–1013. https://doi.org/10.1016/j.eswa.2012.08.013

John A, Paraskevadakis D, Bury A, Yang Z, Riahi R (2014) An integrated fuzzy risk assessment for seaport operations. Saf Sci 68:180–194. https://doi.org/10.1016/j.ssci.2014.04.001

Kanani-Sadat Y, Arabsheibani R, Karimipour F, Nasseri M (2019) A new approach to flood susceptibility assessment in data-scarce and un-gauged regions based on GIS-based hybrid multi criteria decision-making method. J Hydrol 572:17–31. https://doi.org/10.1016/j.jhydrol.2019.02.034

KobryńA, Prystrom J (2016) A data pre-processing model for the TOPSIS method. Folia Oeconomica Stetinensia 16:219–235. https://doi.org/10.1515/foli-2016-0036

Latinopoulos D, Kechagia K (2015) A GIS-based multi-criteria evaluation for wind farm site selection. A regional scale application in Greece. Renew Energy 78:550–560. https://doi.org/10.1016/j.renene.2015.01.041

Le Cozannet G, Garcin M, Bulteau T, Mirgon C, Yates ML, Méndez M, Baills A, Idier D, Oliveros C (2013) An AHP-derived method for mapping the physical vulnerability of coastal areas at regional scales. Nat Hazards Earth Syst Sci 13:1209–1227. https://doi.org/10.5194/nhess-13-1209-2013

Liu C, Frazier P, Kumar L, Macgregor C, Blake N (2006) Catchment-Wide wetland assessment and prioritization using the Multi-Criteria Decision-Making method TOPSIS. Environ Manag 38:316–326. https://doi.org/10.1007/s00267-005-0151-0

Lupo T (2015) Fuzzy Serv Perf model combined with ELECTRE III to comparatively evaluate service quality of international airports in Sicily. J Air Transp Manag 42:249–259

Nautiyal MD (1994) Morphometric analysis of a drainage basin using aerial photographs: A case study of Khairkuli basin, district Dehradun, U.P.. J Indian Soc Remote Sens 22:251–261. https://doi.org/10.1007/BF03026526

Mahmoud SH, Gan TY (2018) Multi-criteria approach to develop flood susceptibility maps in arid regions of Middle East. J Clean Prod 196:216–229. https://doi.org/10.1016/j.jclepro.2018.06.047

Malekian A, Azarnivand A (2016) Application of integrated Shannon's entropy and VIKOR techniques in prioritization of flood risk in the Shemshak watershed, Iran. Water Resour Manage 30:409–425. https://doi.org/10.1007/s11269-015-1169-6

Malekian A, Azarniv A (2016) Application of integrated Shannon's entropy and VIKOR techniques in prioritization of flood risk in the Shemshak watershed. Iran. Water Resour Manag 30:409–425. https://doi.org/10.1007/s11269-015-1169-6

Meshram SG, Sharma SK (2017) Prioritization of watershed through morphometric parameters: a PCA-based approach. Appl Water Sci 7:1505–1519. https://doi.org/10.1007/s13201-015-0332-9

Meshram SG, Alvandi E, Singh VP, Meshram C (2019) Comparison of AHP and fuzzy AHP models for prioritization of watersheds. Soft Comput 23(24):13615–13625. https://doi.org/10.1007/s00500-019-03900-z

Meshram SG, Alvandi E, Meshram C, Kahya E, Al-Quraishi AMD (2020a) Application of SAW and TOPSIS in prioritizing watersheds. Water Resour Manag 34:715–732

Meshram SG, Singh VP, Kahya E, Alvandi E, Meshram C, Sharma SK (2020b) The feasibility of multi-criteria decision making approach for prioritization of sensitive area at risk of water erosion. Water Resour Manage 34:4665–4685. https://doi.org/10.1007/s11269-020-02681-7

Meshram SG, Meshram C (2020) An effective dynamic runoff-sediment yield modelling for Shakkar watershed central India. Arab J Geosci 13:1248. https://doi.org/10.1007/s12517-020-06162-4

Meshram SG, Pourghasemi HR, Abba SI, Alvandi E, Meshram C, Khedher KM (2021) A comparative study between dynamic and soft computing models for sediment forecasting. Soft Comput 25:11005–11017. https://doi.org/10.1007/s00500-021-05834-x

Mira MA, Ghazvinei PT, Sulaiman NMN, Basri NEA, Saheri S, Mahmood NZ, Jahan A, Begum RA, Aghamohammadi N (2016) Application of TOPSIS and VIKOR improved versions in a multi criteria decision analysis to develop an optimized municipal solid waste management model. J Environ Manag 166:109–115

Mulliner E, Malys N, Maliene V (2016) Comparative analysis of MCDM methods for the assessment of sustainable housing affordability. Omega 59:146–156

Nguyen H (2016) A new interval-value d knowledge measure for interval-valued intuitionistic fuzzy sets and application in decision making. Expert Syst Appl 56:143–155. https://doi.org/10.1016/j.eswa.2016.03.007

Ozelkan EC, Duckstein L (2001) Fuzzy conceptual rainfall-runoff models. J Hydrol 253:41–68. https://doi.org/10.1016/S0022-1694(01)00430-9

Pamučar D, Mihajlović M, Obradović R, Atanasković P (2017) Novel approach to group multi-criteria decision making based on interval rough numbers: hybrid DEMATEL-ANP-MAIRCA model. Expert Syst Appl 88:58–80. https://doi.org/10.1016/j.eswa.2017.06.037

Pilevar AR, Matinfar HR, Sohrabi A, Sarmadian F (2020) Integrated fuzzy, AHP and GIS techniques for land suitability assessment in semi-arid regions for wheat and maize farming. Ecol Ind 110:105887

Pourghasemi HR, Pradhan B, Gokceoglu C (2012) Application of fuzzy logic and analytical hierarchy process (AHP) to landslide susceptibility mapping at Haraz watershed Iran. Nat Hazards 63:965–996

Pradeep GS, Ninu Krishnan MV, Vijith H (2014) Identification of critical soil erosion prone areas and annual average soil loss in an upland agricultural watershed of Western Ghats, using analytical hierarchy process (AHP) and RUSLE techniques. Arab J Geosci. https://doi.org/10.1007/s12517-014-1460-5

Rahmati O, Tahmasebipour N, Haghizadeh A, Pourghasemi HR, Feizizadeh B (2017) Evaluation of different machine learning models for predicting and mapping the susceptibility of gully erosion. Geomorphology. https://doi.org/10.1016/j.geomorph.2017.09.006

Raj D, Shaji E (2017) Fluoride contamination in groundwater resources of Alleppey. Southern India Geosci Front 8(1):117–124. https://doi.org/10.1016/j.gsf.2016.01.002

Raju KS, Kumar DN, Jalali A (2017) Prioritization of sub-catchments of a river basin using DEM and fuzzy VIKOR. H2 Open J :1–11. https://doi.org/10.2166/h2oj.2017.001

Rao AR, Srinivas VV (2006) Regionalization of watersheds by fuzzy cluster analysis. J Hydrol 318:57–79. https://doi.org/10.1016/j.jhydrol.2005.06.004

Roszkowska E, Wachowicz T (2015) Application of fuzzy TOPSIS to scoring the negotiation offers in ill-structured negotiation problems. Eur J Oper Res 242:920–932

Saaty TL (1980) The analytical hierarchy process. McGraw-Hill, New York.

Saha S, Gayen A, Pourghasemi HR, Tiefenbacher JP (2019) Identification of soil erosion-susceptible areas using fuzzy logic and analytical hierarchy process modeling in an agricultural watershed of Burdwan district. India Environ Earth Sci 78:649. https://doi.org/10.1007/s12665-019-8658-5

Salehi A, Izadikhah M (2014) A novel method to extend SAW for decision-making problems with interval data. Decision Sci Lett 3:225–236

Sepehri M, Ildoromi AR, Malekinezhad H, Hosseini SZ, Talebi A, Goodarzi S (2017) Flood hazard mapping for the gonbad chi region. Iran. J Environ Eng Sci 12(1):16–24. https://doi.org/10.1680/jenes.16.00017

Sepehri M, Malekinezhad H, Hosseini SZ, Ildoromi AR (2019) Suburban flood hazard mapping in Hamadan city, Iran. Paper presented at the proceedings of the institution of civil engineers municipal engineer.

Shahabi H, Hashim M, Ahmad BB (2015) Remote sensing and GIS based landslide susceptibility mapping using frequency ratio, logistic regression, and fuzzy logic methods at the central Zab basin Iran. Environ Earth Sci 73:8647–8668. https://doi.org/10.1007/s12665-015-4028-0

Sharma SK, Seth NK, Tignath S (2011) Geomorphometric study of Gusuru river watershed using remote sensing & GIS technique. JNKVV Res J 45(2):181–187

Shih HS, Shyur HJ, Lee ES (2007) An extension of TOPSIS for group decision making. Math Comput Model 45:801–813. https://doi.org/10.1016/j.mcm.2006.03.023

Shojaie AB, Babaie S, Sayah E, Mohammaditabar D (2017) Analysis and prioritization of green health suppliers using fuzzy ELECTRE method with a case study. Glob J Flex Syst Manag 19:39–52. https://doi.org/10.1007/s40171-017-0168-2

Soltani A, Hewage K, Reza B, Sadiq R (2015) Multiple stakeholders in multi-criteria decision-making in the context of municipal solid waste management: a review. Waste Manage 35:318–328

Souissi D, Zouhri L, Hammami S, Msaddek MH, Zghibi A, Dlala M (2019) GIS-based MCDM-AHP modeling for flood susceptibility mapping of arid areas, southeastern Tunisia. Geocarto Int. https://doi.org/10.1080/10106049.2019.1566405

Strahler AN (1964) Quantitative geomorphology of drainage basins and channel networks. Section 4-II. In: Chow VT (ed) Handbook of applied hydrology. McGraw-Hill, New York, pp 439–476

Suresh KR, Mujumdar PP (2004) A fuzzy risk approach for performance evaluation of an irrigation reservoir system. Agric Water Manag 69:159–177. https://doi.org/10.1016/j.agwat.2004.05.001

Svoray T, Michailov E, Cohen A, Rokah L, Sturm A (2012) Predicting gully initiation: comparing data mining techniques, analytical hierarchy processes and the topographic threshold. Earth Surface Processes Landforms 37(6):607–619. https://doi.org/10.1002/esp.2273

Sen O, Kahya E (2017) Determination of flood risk: A case study in the rainiest city of Turkey. Environ Model Softw 93:296–309. https://doi.org/10.1016/j.envsoft.2017.03.030

Thapa R, Gupta S, Reddy D (2017) Application of geospatial modeling technique in delineation of fluoride contamination zones within Dwarka Basin, Birbhum India. Geosci Front 8(5):1105–1114. https://doi.org/10.1016/j.gsf.2016.11.006

UNEP (1997) Soil degradation, Map, World Atlas of Desertification, International Soil Reference and Information Centre (ISRIC), UNEP/GRID-Arendal

Vahdani B, Tavakkoli-Moghaddam R, Meysam MS (2013) Soft computing based on new interval-valued fuzzy modified multi-criteria decision-making method. Appl Soft Comput 13:165–172. https://doi.org/10.1016/j.asoc.2012.08.020

Vijith H, Dodge-Wan D (2019) Modelling terrain erosion susceptibility of logged and regenerated forested region in northern Borneo through the Analytical Hierarchy Process (AHP) and GIS techniques. Geo-Environ Disasters 6:8. https://doi.org/10.1186/s40677-019-0124-x

Xu X, Law R, Chen W, Tang L (2016) Forecasting tourism demand by extracting fuzzy Takagi-Sugeno rules from trained SVMs. CAAI Trans Intell Technol. 1:30–42. https://doi.org/10.1016/j.trit.2016.03.004

Yu YB, Wang BD, Wang GL, Li W (2004) Multi-person multi-objective fuzzy decision making model for reservoir flood control operation. Water Resour Manage 18:111–124. https://doi.org/10.1023/B:WARM.0000024705.63932.3c

Yu PS, Chen ST (2005) Updating real-time flood forecasting using a fuzzy rule-based model. Hydrol Sci J 50(2):265–278. https://doi.org/10.1623/hysj.50.2.265.61796

Yu W, Zhang Z, Zhong Q, Sun L (2017) Extended TODIM for multi-criteria group decision making based on unbalanced hesitant fuzzy linguistic term sets. Comput Indus Eng 114:316–328

Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353

Zavadskas EK, Turskis Z, Bagočius V (2015) Multi-criteria selection of a deep-water port in the Eastern Baltic Sea. Appl Soft Comput 26:180–192

Zhu B, Xu Z, Zhang R, Hong M (2015) Generalized analytic network process. Eur J Oper Res 244:277–288. https://doi.org/10.1016/j.ejor.2015.01.011

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